22

Integrals

TOPIC 1 Standard Integrals, Integration by Substitution, Integration by Parts

1. $\lim_{x \to 1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$ **[Sep. 06, 2020 (I)]**

(a) is equal to $\frac{1}{2}$ (b) is equal to 1

(c) is equal to $-\frac{1}{2}$ (d) does not exist

2. If $\int (e^{2x}+2e^x-e^{-x}-1)e^{(e^x+e^{-x})}dx = g(x)e^{(e^x+e^{-x})}+c$, where c is a constant of integeration, then $g(0)$ is equal to: **[Sep. 05, 2020 (I)]**

(a) e (b) e^2 (c) 1 (d) 2

3. If $\int \dfrac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta = A\log_e|B(\theta)|+C$, where C is a constant of integration, then $\dfrac{B(\theta)}{A}$ can be : **[Sep. 05, 2020 (II)]**

(a) $\dfrac{2\sin\theta+1}{\sin\theta+3}$ (b) $\dfrac{2\sin\theta+1}{5(\sin\theta+3)}$

(c) $\dfrac{5(\sin\theta+3)}{2\sin\theta+1}$ (d) $\dfrac{5(2\sin\theta+1)}{\sin\theta+3}$

4. The integral $\int\left(\dfrac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to (where C is a constant of integration) : **[Sep. 04, 2020 (I)]**

(a) $\tan x - \dfrac{x\sec x}{x\sin x+\cos x}+C$

(b) $\sec x + \dfrac{x\tan x}{x\sin x+\cos x}+C$

(c) $\sec x - \dfrac{x\tan x}{x\sin x+\cos x}+C$

(d) $\tan x + \dfrac{x\sec x}{x\sin x+\cos x}+C$

5. Let $f(x)=\int\dfrac{\sqrt{x}}{(1+x)^2}dx\,(x\ge 0)$. Then $f(3)-f(1)$ is equal to : **[Sep. 04, 2020 (I)]**

(a) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (b) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

(c) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (d) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

6. If $\int \sin^{-1}\left(\sqrt{\dfrac{x}{1+x}}\right)dx = A(x)\tan^{-1}(\sqrt{x})+B(x)+C$, where C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be : **[Sep. 03, 2020 (II)]**

(a) $(x+1, -\sqrt{x})$ (b) $(x+1, \sqrt{x})$

(c) $(x-1, -\sqrt{x})$ (d) $(x-1, \sqrt{x})$

7. The integral $\int\dfrac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ is equal to: (where C is a constant of integration) **[Jan. 9, 2020 (I)]**

(a) $\left(\dfrac{x-3}{x+4}\right)^{1/7}+C$ (b) $-\left(\dfrac{x-3}{x+4}\right)^{-1/7}+C$

(c) $\dfrac{1}{2}\left(\dfrac{x-3}{x+4}\right)^{3/7}+C$ (d) $-\dfrac{1}{13}\left(\dfrac{x-3}{x+4}\right)^{-13/7}+C$

8. If $\int\dfrac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)} = \lambda\tan\theta+2\log_e|f(\theta)|+C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to: **[Jan. 9, 2020 (II)]**

(a) $(1, 1-\tan\theta)$ (b) $(-1, 1-\tan\theta)$

(c) $(-1, 1+\tan\theta)$ (d) $(1, 1+\tan\theta)$

9. If $\int\dfrac{\cos x\,dx}{\sin^3 x(1+\sin^6 x)^{2/3}} = f(x)(1+\sin^6 x)^{1/\lambda}+c$ where c is a constant of integration, then $\lambda f\left(\dfrac{\pi}{3}\right)$ is equal to: **[Jan. 8, 2020 (II)]**

(a) $-\frac{9}{8}$ (b) 2 (c) $\frac{9}{8}$ (d) -2

10. The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to :
(Here C is a constant of integration) **[April 12, 2019 (I)]**

(a) $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2}+C$ (b) $\frac{1}{2}\log_e \frac{(x^3+1)^2}{|x^3|}+C$

(c) $\log_e \left|\frac{x^3+1}{x}\right|+C$ (d) $\log_e \frac{|x^3+1|}{x^2}+C$

11. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\int \frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx = A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$, where C is a constant of integration, then the functions A(x) and B(x) are respectively : **[April 12, 2019 (II)]**

(a) $x+\alpha$ and $\log_e|\sin(x+\alpha)|$

(b) $x-\alpha$ and $\log_e|\sin(x-\alpha)|$

(c) $x-\alpha$ and $\log_e|\cos(x-\alpha)|$

(d) $x+\alpha$ and $\log_e|\sin(x-\alpha)|$

12. If $\int \frac{dx}{(x^2-2x+10)^2}$

$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{f(x)}{x^2-2x+10}\right)+C$ where C is a constant of integration, then : **[April 10, 2019 (I)]**

(a) $A=\frac{1}{54}$ and $f(x)=3(x-1)$

(b) $A=\frac{1}{81}$ and $f(x)=3(x-1)$

(c) $A=\frac{1}{27}$ and $f(x)=9(x-1)$

(d) $A=\frac{1}{54}$ and $f(x)=9(x-1)^2$

13. If f(x) is a non-zero polynomial of degree four, having local extreme points at x = –1, 0, 1; then the set $S=\{x \in R : f(x)=f(0)\}$ contains exactly: **[April 09, 2019 (I)]**

(a) four irrational numbers.
(b) four rational numbers.
(c) two irrational and two rational numbers.
(d) two irrational and one rational number.

14. The integral $\int \sec^{2/3} x \ \mathrm{cosec}^{4/3} x dx$ is equal to: **[April 09, 2019 (I)]**

(a) $-3\tan^{-1/3} x + C$ (b) $-\frac{3}{4}\tan^{-4/3} x + C$

(c) $-3\cot^{-1/3} x + C$ (d) $3\tan^{-1/3} x + C$

(Here C is a constant of integration)

15. If $\int e^{\sec x}(\sec x\tan x\, f(x)+(\sec x\tan x+\sec^2 x))\,dx = e^{\sec x} f(x) + C$, then a possible choice of $f(x)$ is: **[April 09, 2019 (II)]**

(a) $\sec x+\tan x+\frac{1}{2}$ (b) $\sec x-\tan x-\frac{1}{2}$

(c) $\sec x+x\tan x-\frac{1}{2}$ (d) $x\sec x+\tan x+\frac{1}{2}$

16. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to :

(where c is a constant of integration.) **[April 08, 2019 (I)]**

(a) $2x+\sin x+2\sin 2x+c$ (b) $x+2\sin x+2\sin 2x+c$
(c) $x+2\sin x+\sin 2x+c$ (d) $2x+\sin x+\sin 2x+c$

17. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}}+C$, where C is a constant of integration, then the function $f(x)$ is equal to : **[April 08, 2019 (II)]**

(a) $\frac{3}{x^2}$ (b) $-\frac{1}{6x^3}$ (c) $-\frac{1}{2x^2}$ (d) $-\frac{1}{2x^3}$

18. The integral $\int \cos(\log_e x)dx$ is equal to :
(where C is a constant of integration) **[Jan. 12, 2019 (I)]**

(a) $\frac{x}{2}[\sin(\log_e x)-\cos(\log_e x)]+C$

(b) $x[\cos(\log_e x)+\sin(\log_e x)]+C$

(c) $\frac{x}{2}[\cos(\log_e x)+\sin(\log_e x)]+C$

(d) $x[\cos(\log_e x)-\sin(\log_e x)]+C$

19. The integral $\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$ is equal to:
(where C is a constant of integration) **[Jan. 12, 2019 (II)]**

(a) $\frac{x^4}{6(2x^4+3x^2+1)^3}+C$ (b) $\frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$

(c) $\frac{x^4}{(2x^4+3x^2+1)^3}+C$ (d) $\frac{x^{12}}{(2x^4+3x^2+1)^3}+C$

20. If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A (x), where C is a constant of integration, then $(A(x))^m$ equals : **[Jan. 11, 2019 (I)]**

(a) $\frac{-1}{27x^9}$ (b) $\frac{-1}{3x^3}$ (c) $\frac{1}{27x^6}$ (d) $\frac{1}{9x^4}$

21. If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1}+C$, where C is a constant of integration, then $f(x)$ is equal to: **[Jan. 11, 2019 (II)]**

(a) $\frac{1}{3}(x+1)$ (b) $\frac{2}{3}(x+2)$

(c) $\frac{2}{3}(x-4)$ (d) $\frac{1}{3}(x+4)$

22. Let $n \ge 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$

Then $\int \frac{(\sin^n\theta + \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$ is equal to: **[Jan 10, 2019(I)]**

(a) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(b) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(c) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(d) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

(where C is a constant of integration)

23. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral **[Jan. 09, 2019(I)]**

$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}dx$ is equal to:

(a) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$

(b) $\frac{1}{2}\log_e|\sec(x^2-1)|+c$

(c) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(d) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$

(where c is a constant of integration)

24. If $f(x)=\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx, (x \ge 0)$, and $f(0)=0$, then the value of $f(1)$ is: **[Jan. 09, 2019 (II)]**

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

25. The integral

$\int\frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$ is equal to: **[2018]**

(a) $\frac{-1}{3(1+\tan^3 x)}+C$ (b) $\frac{1}{1+\cot^3 x}+C$

(c) $\frac{-1}{1+\cot^3 x}+C$ (d) $\frac{1}{3(1+\tan^3 x)}+C$

(where C is a constant of integration)

26. If

$\int\frac{\tan x}{1+\tan x+\tan^2 x}dx = x-\frac{K}{\sqrt{A}}\tan^{-1}\left(\frac{K\tan x+1}{\sqrt{A}}\right)+C,$

(C is a constant of integration), then the ordered pair (K, A) is euqal to **[Online April 16, 2018]**

(a) (2, 3) (b) (2, 1) (c) (–2, 1) (d) (–2, 3)

27. If $f\left(\frac{x-4}{x+2}\right)=2x+1, (x \in R = \{1,-2\})$, then $\int f(x)\,dx$ is equal to (where C is a constant of integration) **[Online April 15, 2018]**

(a) $12\log_e|1-x|-3x+c$
(b) $-12\log_e|1-x|-3x+c$
(c) $-12\log_e|1-x|+3x+c$
(d) $12\log_e|1-x|+3x+c$

28. $\int\frac{2x+5}{\sqrt{7-6x-x^2}}dx = A\sqrt{7-6x-x^2}+B\sin^{-1}\left(\frac{x+3}{4}\right)+C$

(where C is a constant of integration), then the ordered pair (A, B) is equal to **[Online April 15, 2018]**

(a) (–2, –1) (b) (2, –1)
(c) (–2, 1) (d) (2, 1)

29. If $f\left(\frac{3x-4}{3x+4}\right)=x+2, x \neq -\frac{4}{3}$, and

$\int f(x)\,dx = A\log|1-x|+Bx+C$, then the ordered pair (A, B) is equal to : **[Online April 9, 2017]**
(where C is a constant of integration)

(a) $\left(\frac{8}{3},\frac{2}{3}\right)$ (b) $\left(-\frac{8}{3},\frac{2}{3}\right)$

(c) $\left(-\frac{8}{3},-\frac{2}{3}\right)$ (d) $\left(\frac{8}{3},-\frac{2}{3}\right)$

30. The integral $\int\sqrt{1+2\cot x(\operatorname{cosec} x+\cot x)}dx$

$\left(0 < x < \frac{\pi}{2}\right)$ is equal to : **[Online April 8, 2017]**

(where C is a constant of integration)

(a) $2\log\left|\sin\frac{x}{2}\right|+C$ (b) $4\log\left|\sin\frac{x}{2}\right|+C$

(c) $2\log\left|\cos\frac{x}{2}\right|+C$ (d) $4\log\left|\cos\frac{x}{2}\right|+C$

31. If $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = (\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then A + B + C equals : **[Online April 9, 2016]**

(a) $\frac{16}{5}$ (b) $\frac{27}{10}$ (c) $\frac{7}{10}$ (d) $\frac{21}{5}$

32. If $\int \frac{\log(t+\sqrt{1+t^2})}{\sqrt{1+t^2}}dt = \frac{1}{2}(g(t))^2 + C$, where C is a constant, then g(b) is equal to : **[Online April 11, 2015]**

(a) $\frac{1}{\sqrt{5}}\log(2+\sqrt{5})$ (b) $\frac{1}{2}\log(2+\sqrt{5})$

(c) $2\log(2+\sqrt{5})$ (d) $\log(2+\sqrt{5})$

33. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to **[2014]**

(a) $(x+1)e^{x+\frac{1}{x}}+c$ (b) $-xe^{x+\frac{1}{x}}+c$

(c) $(x-1)e^{x+\frac{1}{x}}+c$ (d) $xe^{x+\frac{1}{x}}+c$

34. The integral $\int \frac{\sin^2 x\cos^2 x}{\left(\sin^3 x+\cos^3 x\right)^2}dx$ is equal to: **[Online April 12, 2014]**

(a) $\frac{1}{\left(1+\cot^3 x\right)}+c$ (b) $-\frac{1}{3\left(1+\tan^3 x\right)}+c$

(c) $\frac{\sin^3 x}{\left(1+\cos^3 x\right)}+c$ (d) $-\frac{\cos^3 x}{3\left(1+\sin^3 x\right)}+c$

35. The integral $\int x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx\ (x>0)$ is equal to: **[Online April 11, 2014]**

(a) $-x+(1+x^2)\tan^{-1}x+c$
(b) $x-(1+x^2)\cot^{-1}x+c$
(c) $-x+(1+x^2)\cot^{-1}x+c$
(d) $x-(1+x^2)\tan^{-1}x+c$

36. $\int \frac{\sin^8 x-\cos^8 x}{\left(1-2\sin^2 x\cos^2 x\right)}dx$ is equal to: **[Online April 9, 2014]**

(a) $\frac{1}{2}\sin 2x+c$ (b) $-\frac{1}{2}\sin 2x+c$

(c) $-\frac{1}{2}\sin x+c$ (d) $-\sin^2 x+c$

37. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to **[2013]**

(a) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^2\psi(x^3)dx\right]+C$

(b) $\frac{1}{3}x^3\psi(x^3)-3\int x^3\psi(x^3)dx+C$

(c) $\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$

(d) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^3\psi(x^3)dx\right]+C$

38. If the integral

$$\int \frac{\cos 8x+1}{\cot 2x-\tan 2x}dx = A\cos 8x+k,$$

where k is an arbitrary constant, then A is equal to : **[Online April 25, 2013]**

(a) $-\frac{1}{16}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

39. If the $\int \frac{5\tan x}{\tan x-2}dx = x+a\ln|\sin x-2\cos x|+k$, then a is equal to : **[2012]**

(a) -1 (b) -2 (c) 1 (d) 2

40. If $f(x)=\int\left(\frac{x^2+\sin^2 x}{1+x^2}\right)\sec^2 x\,dx$ and $f(0)=0$, then $f(1)$ equals **[Online May 19, 2012]**

(a) $\tan 1-\frac{\pi}{4}$ (b) $\tan 1+1$

(c) $\frac{\pi}{4}$ (d) $1-\frac{\pi}{4}$

41. The integral of $\frac{x^2-x}{x^3-x^2+x-1}$ w.r.t. x is **[Online May 12, 2012]**

(a) $\frac{1}{2}\log\left(x^2+1\right)+C$ (b) $\frac{1}{2}\log\left|x^2-1\right|+C$

(c) $\log\left(x^2+1\right)+C$ (d) $\log\left|x^2-1\right|+C$

42. Let $f(x)$ be an indefinite integral of $\cos^3 x$.

Statement 1: $f(x)$ is a periodic function of period π.

Statement 2: $\cos^3 x$ is a periodic function.

[Online May 7, 2012]

(a) Statement 1 is true, Statement 2 is false.
(b) Both the Statements are true, but Statement 2 is not the correct explanation of Statement 1.
(c) Both the Statements are true, and Statement 2 is correct explanation of Statement 1.
(d) Statement 1 is false, Statement 2 is true.

43. The value of $\sqrt{2}\int \frac{\sin x dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is **[2008]**

(a) $x+\log|\cos\left(x-\frac{\pi}{4}\right)|+c$

(b) $x-\log|\sin\left(x-\frac{\pi}{4}\right)|+c$

(c) $x+\log|\sin\left(x-\frac{\pi}{4}\right)|+c$

(d) $x-\log|\cos\left(x-\frac{\pi}{4}\right)|+c$

44. $\int\frac{dx}{\cos x+\sqrt{3}\sin x}$ equals **[2007]**

(a) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(c) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(d) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

45. $\int\frac{dx}{\cos x-\sin x}$ is equal to **[2004]**

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

(b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

46. If $\int\frac{\sin x}{\sin(x-\alpha)}dx = Ax+B\log\sin(x-\alpha),+C,$ then value of (A, B) is **[2004]**

(a) $(-\cos\alpha,\sin\alpha)$ (b) $(\cos\alpha,\sin\alpha)$

(c) $(-\sin\alpha,\cos\alpha)$ (d) $(\sin\alpha,\cos\alpha)$

47. $f(x)$ and $g(x)$ are two differentiable functions on [0, 2] such that $f''(x)-g''(x)=0,\ f'(1)=2g'(1)=4f(2)=3g(2)=9$ then $f(x)-g(x)$ at $x=3/2$ is **[2002]**

(a) 0 (b) 2 (c) 10 (d) 5

TOPIC 2 Integration of the Forms: ∫ex(f(x) + f'(x))dx, ∫ekx(df(x) + f'(x))dx, Integration by Partial Fractions, Integration of Some Special Irrational Algebraic Functions, Integration of Different Expressions of e^x

48. The integral $\int_1^2 e^x\cdot x^x(2+\log_e x)\,dx$ equals: **[Sep. 06, 2020 (II)]**

(a) $e(4e+1)$ (b) $4e^2-1$

(c) $e(4e-1)$ (d) $e(2e-1)$

49. A value of α such that

$$\int_{\alpha}^{\alpha+1}\frac{dx}{(x+\alpha)(x+\alpha+1)}=\log_e\left(\frac{9}{8}\right)$$ is : **[April 12, 2019 (II)]**

(a) -2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 2

50. If $\int x^5e^{-x^2}dx = g(x)e^{-x^2}+c$, where c is a constant of integration, then $g(-1)$ is equal to : **[April 10, 2019 (II)]**

(a) -1 (b) 1 (c) $-\frac{5}{2}$ (d) $-\frac{1}{2}$

51. If $\int x^5 e^{-4x^3}dx=\frac{1}{48}e^{-4x^3}f(x)+C$, where C is a constant of integration, then $f(x)$ is equal to: **[Jan. 10, 2019 (II)]**

(a) $-2x^3-1$ (b) $-4x^3-1$

(c) $-2x^3+1$ (d) $4x^3+1$

52. The integral $\int\frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ is equal to :

(where C is a constant of integration) **[Online April 10, 2016]**

(a) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$ (b) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$

(c) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$ (d) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

53. The integral $\int\frac{dx}{x^2(x^4+1)^{3/4}}$ equals : **[2015]**

(a) $-(x^4+1)^{\frac{1}{4}}+c$ (b) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ (d) $(x^4+1)^{\frac{1}{4}}+c$

54. The integral $\int \frac{dx}{(x+1)^{\frac{3}{4}}(x-2)^{\frac{5}{4}}}$ is equal to : **[Online April 10, 2015]**

(a) $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}}+C$ (b) $4\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}}+C$

(c) $4\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}}+C$ (d) $-\frac{4}{3}\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}}+C$

55. If m is a non-zero number and

$$\int \frac{x^{5m-1}+2x^{4m-1}}{\left(x^{2m}+x^{m}+1\right)^{3}}dx = f(x)+c,$$

then f(x) is: **[Online April 19, 2014]**

(a) $\frac{x^{5m}}{2m\left(x^{2m}+x^{m}+1\right)^{2}}$ (b) $\frac{x^{4m}}{2m\left(x^{2m}+x^{m}+1\right)^{2}}$

(c) $\frac{2m\left(x^{5m}+x^{4m}\right)}{\left(x^{2m}+x^{m}+1\right)^{2}}$ (d) $\frac{\left(x^{5m}-x^{4m}\right)}{2m\left(x^{2m}+x^{m}+1\right)^{2}}$

56. The integral $\int \frac{x\,dx}{2-x^{2}+\sqrt{2-x^{2}}}$ equals : **[Online April 23, 2013]**

(a) $\log\left|1+\sqrt{2+x^{2}}\right|+c$ (b) $-\log\left|1+\sqrt{2-x^{2}}\right|+c$

(c) $-x\log\left|1-\sqrt{2-x^{2}}\right|+c$ (d) $x\log\left|1-\sqrt{2+x^{2}}\right|+c$

57. If $\int \frac{x^{2}-x+1}{x^{2}+1}e^{\cot^{-1}x}dx = A(x)e^{\cot^{-1}x}+C$, then $A(x)$ is equal to : **[Online April 22, 2013]**

(a) $-x$ (b) x (c) $\sqrt{1-x}$ (d) $\sqrt{1+x}$

58. If $\int \frac{dx}{x+x^{7}} = p(x)$ then, $\int \frac{x^{6}}{x+x^{7}}dx$ is equal to: **[Online April 9, 2013]**

(a) $\ln|x|-p(x)+c$ (b) $\ln|x|+p(x)+c$
(c) $x-p(x)+c$ (d) $x+p(x)+c$

59. $\int \left\{\frac{(\log x-1)}{1+(\log x)^{2}}\right\}^{2} dx$ is equal to **[2005]**

(a) $\frac{\log x}{(\log x)^{2}+1}+C$ (b) $\frac{x}{x^{2}+1}+C$

(c) $\frac{xe^{x}}{1+x^{2}}+C$ (d) $\frac{x}{(\log x)^{2}+1}+C$

TOPIC 3 Evaluation of Definite Integral by Substitution, Properties of Definite Integrals

60. If $I_1 = \int_0^1 (1-x^{50})^{100}dx$ and $I_2 = \int_0^1 (1-x^{50})^{101}dx$ such that $I_2 = \alpha I_1$ then α equals to : **[Sep. 06, 2020 (I)]**

(a) $\frac{5049}{5050}$ (b) $\frac{5050}{5049}$ (c) $\frac{5050}{5051}$ (d) $\frac{5051}{5050}$

61. The value of $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}dx$ is: **[Sep. 05, 2020 (I)]**

(a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

62. Let $f(x)=|x-2|$ and $g(x)=f(f(x))$, $x\in[0, 4]$.

Then $\int_0^3 (g(x)-f(x))\,dx$ is equal to : **[Sep. 04, 2020 (I)]**

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

63. The integral

$$\int_{\pi/6}^{\pi/3} \tan^{3}x\cdot\sin^{2}3x(2\sec^{2}x\cdot\sin^{2}3x+3\tan x\cdot\sin 6x)\,dx$$

is equal to : **[Sep. 04, 2020 (II)]**

(a) $\frac{7}{18}$ (b) $-\frac{1}{9}$ (c) $-\frac{1}{18}$ (d) $\frac{9}{2}$

64. Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\le x$ respectively of a real number x. If $\int_0^n \{x\}\,dx$, $\int_0^n [x]\,dx$ and $10(n^2-n)$, $(n\in\mathbf{N}, n>1)$ are three consecutive terms of a G.P., then n is equal to ________________. **[NA Sep. 04, 2020 (II)]**

65. $\int_{-\pi}^{\pi} |\pi-|x||\,dx$ is equal to : **[Sep. 03, 2020 (I)]**

(a) $\sqrt{2}\pi^{2}$ (b) $2\pi^{2}$ (c) π^{2} (d) $\frac{\pi^{2}}{2}$

66. If the value of the integral $\int_0^{1/2} \frac{x^{2}}{(1-x^{2})^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to : **[Sep. 03, 2020 (II)]**

(a) $2\sqrt{3}-\pi$ (b) $2\sqrt{3}+\pi$
(c) $3\sqrt{2}+\pi$ (d) $3\sqrt{2}-\pi$

67. The integral $\int_0^2 \left||x-1|-x\right|dx$ is equal to _________. **[NA Sep. 02, 2020 (I)]**

68. Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|\,dx$ is ________.
[NA Sep. 02, 2020 (II)]

69. If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$; then $\int_0^1 f(x)dx$ is equal to: **[Jan. 9, 2020 (I)]**

(a) $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$ (b) $\frac{1}{2}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$

(c) $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$ (d) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

70. The value of $\int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$ is equal to:
[Jan. 9, 2020 (I)]

(a) 2π (b) $2\pi^2$ (c) π^2 (d) 4π

71. If $I=\int_1^2\frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then: **[Jan. 8, 2020 (II)]**

(a) $\frac{1}{8}<I^2<\frac{1}{4}$ (b) $\frac{1}{9}<I^2<\frac{1}{8}$

(c) $\frac{1}{16}<I^2<\frac{1}{9}$ (d) $\frac{1}{6}<I^2<\frac{1}{2}$

72. If $f(a+b+1-x)=f(x)$, for all x, where a and b are fixed positive real numbers, then $\frac{1}{a+b}\int_a^b x(f(x)+f(x+1))dx$ is equal to: **[Jan. 7, 2020 (I)]**

(a) $\int_{a+1}^{b+1} f(x)dx$ (b) $\int_{a-1}^{b-1} f(x)dx$

(c) $\int_{a-1}^{b-1} f(x+1)dx$ (d) $\int_{a+1}^{b+1} f(x+1)dx$

73. The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx=5$, is :
[Jan. 7, 2020 (II)]

(a) $\log_e 2$ (b) $\log_e\left(\frac{3}{2}\right)$ (c) $\log_e\sqrt{2}$ (d) $\log_e\left(\frac{4}{3}\right)$

74. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0,2\pi)-\{\pi\}$ which satisfy the equation, $2\cot^2\theta-\frac{5}{\sin\theta}+4=0$, then $\int_{\theta_1}^{\theta_2}\cos^2 3\theta\,d\theta$ is equal to:
[Jan. 7, 2020 (II)]

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{3}+\frac{1}{6}$ (d) $\frac{\pi}{9}$

75. Let $f: R\to R$ be a continuously differentiable function such that $f(2)=6$ and $f'(2)=\frac{1}{48}$.
If $\int_6^{f(x)} 4t^3dt=(x-2)g(x)$, then $\lim_{x\to 2} g(x)$ is equal to :
[April 12, 2019 (I)]

(a) 18 (b) 24 (c) 12 (d) 36

76. If $\int_0^{\frac{x}{2}}\frac{\cot x}{\cot x+\text{cosec}\,x}dx=m(\pi+n)$, then m.n is equal to :
[April 12, 2019 (I)]

(a) $-\frac{1}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) –1

77. The value of $\int_0^{2\pi}[\sin 2x(1+\cos 3x)]dx$, where [t] denotes the greatest integer function, is: **[April 10, 2019 (I)]**

(a) π (b) –π (c) –2π (d) 2π

78. The integral $\int_{\pi/6}^{\pi/3}\sec^{2/3}x\,\text{cosec}^{4/3}x\,dx$ is equal to :
[April 10, 2019 (II)]

(a) $3^{5/6}-3^{2/3}$ (b) $3^{4/3}-3^{1/3}$

(c) $3^{7/6}-3^{5/6}$ (d) $3^{5/3}-3^{1/3}$

79. The value of $\int_0^{\pi/2}\frac{\sin^3 x}{\sin x+\cos x}dx$ is: **[April 9, 2019 (I)]**

(a) $\frac{\pi-2}{8}$ (b) $\frac{\pi-1}{4}$ (c) $\frac{\pi-2}{4}$ (d) $\frac{\pi-1}{2}$

80. The value of the integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)dx$ is:
[April 09, 2019 (II)]

(a) $\frac{\pi}{2}-\frac{1}{2}\log_e 2$ (b) $\frac{\pi}{4}-\log_e 2$

(c) $\frac{\pi}{2}-\log_e 2$ (d) $\frac{\pi}{4}-\frac{1}{2}\log_e 2$

81. If $f: R\to R$ is a differentiable function and $f(2)=6$, then $\lim_{x\to 2}\int_6^{f(x)}\frac{2t\,dt}{(x-2)}$ is: **[April 09, 2019 (II)]**

(a) $24f'(2)$ (b) $2f'(2)$ (c) 0 (d) $12f'(2)$

82. If $f(x)=\frac{2-x\cos x}{2+x\cos x}$ and $g(x)=\log_e x$, (x > 0) then the value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x))dx$ is :
[April 8, 2019 (I)]

(a) $\log_e 3$ (b) $\log_e e$ (c) $\log_e 2$ (d) $\log_e 1$

83. Let $f(x)=\int_0^x g(t)\,dt$, where g is a non-zero even function. If $f(x+5)=g(x)$, then $\int_0^x f(t)\,dt$ equals : **[April 08, 2019 (II)]**

(a) $\int_{x+5}^{5} g(t)\,dt$ (b) $\int_{5}^{x+5} g(t)\,dt$

(c) $2\int_{5}^{x+5} g(t)\,dt$ (d) $5\int_{x+5}^{5} g(t)\,dt$

84. Let f and g be continuous functions on [0, a] such that $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$, then $\int_0^a f(x)g(x)\,dx$ is equal to: **[Jan. 12, 2019 (I)]**

(a) $4\int_0^a f(x)\,dx$ (b) $\int_0^a f(x)\,dx$

(c) $2\int_0^a f(x)\,dx$ (d) $-3\int_0^a f(x)\,dx$

85. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{x}\right\}\log_e x\,dx$ is equal to : **[Jan. 12, 2019 (II)]**

(a) $\frac{1}{2}-e-\frac{1}{e^2}$ (b) $-\frac{1}{2}+\frac{1}{e}-\frac{1}{2e^2}$

(c) $\frac{3}{2}-\frac{1}{e}-\frac{1}{2e^2}$ (d) $\frac{3}{2}-e-\frac{1}{2e^2}$

86. The value of the integral $\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$ (where [x] denotes the greatest integer less than or equal to x) is : **[Jan. 11, 2019 (I)]**

(a) 0 (b) sin 4 (c) 4 (d) 4 – sin 4

87. The integral $\int_{\pi/6}^{\pi/4}\frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$ equals : **[Jan. 11, 2019 (II)]**

(a) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$ (b) $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(c) $\frac{\pi}{40}$ (d) $\frac{1}{5}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

88. Let $I=\int_a^b (x^4-2x^2)dx$. If I is minimum then the ordered pair (a, b) is: **[Jan 10, 2019 (I)]**

(a) $(0,\sqrt{2})$ (b) $(-\sqrt{2},0)$

(c) $(\sqrt{2},-\sqrt{2})$ (d) $(-\sqrt{2},\sqrt{2})$

89. If $\int_0^x f(t)\,dt = x^2+\int_x^1 t^2 f(t)\,dt$, then $f'(1/2)$ is: **[Jan. 10, 2019 (II)]**

(a) $\frac{24}{25}$ (b) $\frac{18}{25}$ (c) $\frac{4}{5}$ (d) $\frac{6}{25}$

90. The value of $\int_{-\pi/2}^{\pi/2}\frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is: **[Jan. 10, 2019 (II)]**

(a) $\frac{1}{12}(7\pi+5)$ (b) $\frac{1}{12}(7\pi-5)$

(c) $\frac{3}{20}(4\pi-3)$ (d) $\frac{3}{10}(4\pi-3)$

91. The value of $\int_0^{\pi}|\cos x|^3\,dx$ is: **[Jan 9, 2019 (I)]**

(a) 0 (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{4}{3}$

92. Let f be a differentiable function from **R** to **R** such that $|f(x)-f(y)|\le 2|x-y|^{3/2}$, for all $x, y, \in$ **R**. If $f(0)=1$ then $\int_0^1 f^2(x)\,dx$ is equal to : **[Jan. 09, 2019 (II)]**

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

93. If $\int_0^{\pi/3}\frac{\tan\theta}{\sqrt{2k\sec\theta}}d\theta=1-\frac{1}{\sqrt{2}},(k>0)$ then the value of k is: **[Jan. 09, 2019 (II)]**

(a) 4 (b) $\frac{1}{2}$ (c) 1 (d) 2

94. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin^2 x}{1+2^x}dx$ is : **[2018]**

(a) $\frac{\pi}{2}$ (b) 4π (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

95. If $f(x)=\int_0^x t(\sin x-\sin t)\,dt$ then **[Online April 16, 2018]**

(a) $f'''(x)+f'(x)=\cos x-2x\sin x$
(b) $f'''(x)+f''(x)-f'(x)=\cos x$
(c) $f'''(x)-f''(x)=\cos x-2x\sin x$
(d) $f'''(x)+f''(x)=\sin x$

96. The value of integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}dx$ is **[Online April 15, 2018]**

(a) $\frac{\pi}{2}(\sqrt{2}+1)$ (b) $\pi(\sqrt{2}-1)$

(c) $2\pi(\sqrt{2}-1)$ (d) $\pi\sqrt{2}$

97. If $I_1=\int_0^1 e^{-x}\cos^2 x\,dx$; $I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx$ and $I_3=\int_0^1 e^{-x^3}dx$; then **[Online April 15, 2018]**

(a) $I_2>I_3>I_1$ (b) $I_3>I_1>I_2$

(c) $I_2>I_1>I_3$ (d) $I_3>I_2>I_1$

98. The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx$ is **[Online April 15, 2018]**

(a) $\frac{3}{16}\pi$ (b) 0 (c) $\frac{3}{8}\pi$ (d) $\frac{3}{4}$

99. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to : **[2017]**

(a) –1 (b) –2 (c) 2 (d) 4

100. Let $I_n=\int \tan^n x\,dx, (n>1)$. $I_4+I_6=a\tan^5 x+bx^5+C$, where C is constant of integration, then the ordered pair (a, b) is equal to : **[2017]**

(a) $\left(-\frac{1}{5},0\right)$ (b) $\left(-\frac{1}{5},1\right)$ (c) $\left(\frac{1}{5},0\right)$ (d) $\left(\frac{1}{5},-1\right)$

101. If $\int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}}=\frac{k}{k+5}$ then k is equal to : **[Online April 9, 2017]**

(a) 1 (b) 2 (c) 3 (d) 4

102. The integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x}{(\tan x+\cot x)^3}dx$ equals : **[Online April 8, 2017]**

(a) $\frac{15}{128}$ (b) $\frac{15}{64}$ (c) $\frac{13}{32}$ (d) $\frac{15}{256}$

103. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to : **[2016]**

(a) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$ (b) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(c) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$ (d) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

104. For $x\in R, x\neq 0$, if y(x) is a differentiable function such that $x\int_1^x y(t)dt=(x+1)\int_1^x ty(t)dt$, then y(x) equals : (where C is a constant) **[Online April 10, 2016]**

(a) $Cx^3e^{\frac{1}{x}}$ (b) $\frac{C}{x^2}e^{-\frac{1}{x}}$ (c) $\frac{C}{x}e^{-\frac{1}{x}}$ (d) $\frac{C}{x^3}e^{-\frac{1}{x}}$

105. The value of the integral

$\int_4^{10}\frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$, where [x] denotes the greatest integer less than or equal to x, is : **[Online April 10, 2016]**

(a) $\frac{1}{3}$ (b) 6 (c) 7 (d) 3

106. If $2\int_0^1 \tan^{-1}x\,dx=\int_0^1 \cot^{-1}(1-x+x^2)dx$, then $\int_0^1 \tan^{-1}(1-x+x^2)dx$ is equal to : **[Online April 9, 2016]**

(a) $\frac{\pi}{2}+\log 2$ (b) log2

(c) $\frac{\pi}{2}-\log 4$ (d) log4

107. The integral **[2015]**

$\int_2^4 \frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$ is equal to :

(a) 1 (b) 6 (c) 2 (d) 4

108. Let $f: R\to R$ be a function such that $f(2-x)=f(2+x)$ and $f(4-x)=f(4+x)$, for all $x\in R$ and $\int_0^2 f(x)\,dx=5$. Then the value of $\int_{10}^{50} f(x)\,dx$ is : **[Online April 11, 2015]**

(a) 125 (b) 80 (c) 100 (d) 200

109. Let $f: (-1, 1)\to R$ be a continuous function. If $\int_0^{\sin x} f(t)dt=\frac{\sqrt{3}}{2}x$, then $f\left(\frac{\sqrt{3}}{2}\right)$ is equal to : **[Online April 11, 2015]**

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{3}$

110. For $x>0$, let $f(x)=\int_1^x \frac{\log t}{1+t}dt$. Then $f(x)+f\left(\frac{1}{x}\right)$ is equal to: **[Online April 10, 2015]**

(a) $\frac{1}{4}(\log x)^2$ (b) $\log x$

(c) $\frac{1}{2}(\log x)^2$ (d) $\frac{1}{4}\log x^2$

111. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals: **[2014]**

(a) $4\sqrt{3}-4$ (b) $4\sqrt{3}-4-\frac{\pi}{3}$

(c) $\pi-4$ (d) $\frac{2\pi}{3}-4-4\sqrt{3}$

112. Let function F be defined as $F(x)=\int_1^x \frac{e^t}{t}dt,\ x>0$ then the value of the integral $\int_1^x \frac{e^t}{t+a}dt$, where $a>0$, is:

[Online April 19, 2014]

(a) $e^a[F(x)-F(1+a)]$

(b) $e^{-a}[F(x+a)-F(a)]$

(c) $e^a[F(x+a)-F(1+a)]$

(d) $e^{-a}[F(x+a)-F(1+a)]$

113. If for a continuous function f(x), $\int_{-\pi}^{t}(f(x)+x)dx=\pi^2-t^2$, for all $t\geq-\pi$, then $f\left(-\frac{\pi}{3}\right)$ is equal to:

[Online April 12, 2014]

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

114. If [ ] denotes the greatest integer function, then the integral $\int_0^{\pi}[\cos x]dx$ is equal to: **[Online April 12, 2014]**

(a) $\frac{\pi}{2}$ (b) 0 (c) –1 (d) $-\frac{\pi}{2}$

115. If for $n\geq 1$, $P_n=\int_1^e(\log x)^n dx$, then $P_{10}-90P_8$ is equal to:

[Online April 11, 2014]

(a) –9 (b) 10e (c) – 9 e (d) 10

116. The integral $\int_0^{\frac{1}{2}}\frac{\ln(1+2x)}{1+4x^2}dx$, equals:

[Online April 9, 2014]

(a) $\frac{\pi}{4}\ln 2$ (b) $\frac{\pi}{8}\ln 2$ (c) $\frac{\pi}{16}\ln 2$ (d) $\frac{\pi}{32}\ln 2$

117. The intercepts on x-axis made by tangents to the curve, $y=\int_0^x|t|dt,\ x\in R$, which are parallel to the line $y=2x$, are equal to : **[2013]**

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

118. Statement-1 : The value of the integral

$$\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}} \text{ is equal to } \pi/6$$

Statement-2 : $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx.$ **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is false; Statement-2 is true.

119. For $0\leq x\leq\frac{\pi}{2}$, the value of

$$\int_0^{\sin^2 x}\sin^{-1}(\sqrt{t})dt+\int_0^{\cos^2 x}\cos^{-1}(\sqrt{t})\,dt \text{ equals :}$$

[Online April 25, 2013]

(a) $\frac{\pi}{4}$ (b) 0 (c) 1 (d) $-\frac{\pi}{4}$

120. The value of $\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx$ is :

[Online April 23, 2013]

(a) π (b) $\frac{\pi}{2}$ (c) 4π (d) $\frac{\pi}{4}$

121. The integral $\int_{7\pi/4}^{7\pi/3}\sqrt{\tan^2 x}\,dx$ is equal to :

[Online April 22, 2013]

(a) $\log 2\sqrt{2}$ (b) $\log 2$

(c) $2\log 2$ (d) $\log\sqrt{2}$

122. If $x=\int_0^y\frac{dt}{\sqrt{1+t^2}}$, then $\frac{d^2y}{dx^2}$ is equal to :

[Online April 9, 2013]

(a) y (b) $\sqrt{1+y^2}$

(c) $\frac{x}{\sqrt{1+y^2}}$ (d) y^2

123. If $g(x)=\int_0^x\cos 4t\,dt$, then $g(x+\pi)$ equals **[2012]**

(a) $\frac{g(x)}{g(\pi)}$ (b) $g(x)+g(\pi)$

(c) $g(x)-g(\pi)$ (d) $g(x).g(\pi)$

124. If $[x]$ is the greatest integer $\le x$, then the value of the integral $\int_{-0.9}^{0.9}\left(\left[x^2\right]+\log\left(\frac{2-x}{2+x}\right)\right)dx$ is **[Online May 26, 2012]**

(a) 0.486 (b) 0.243 (c) 1.8 (d) 0

125. The value of the integral $\int_0^{0.9}\left[x-2[x]\right]dx$,

where [.] denotes the greatest integer function is **[Online May 19, 2012]**

(a) 0.9 (b) 1.8 (c) -0.9 (d) 0

126. If $\frac{d}{dx}G(x)=\frac{e^{\tan x}}{x}$, $x\in(0,\pi/2)$, then

$\int_{1/4}^{1/2}\frac{2}{x}.e^{\tan(\pi x^2)}dx$ is equal to **[Online May 12, 2012]**

(a) $G(\pi/4)-G(\pi/16)$ (b) $2[G(\pi/4)-G(\pi/16)]$

(c) $\pi[G(1/2)-G(1/4)]$ (d) $G(1/\sqrt{2})-G(1/2)$

127. If $\int_e^x t\,f(t)\,dt=\sin x-x\cos x-\frac{x^2}{2}$, for all $x\in R-\{0\}$,

then the value of $f\left(\frac{\pi}{6}\right)$ is **[Online May 7, 2012]**

(a) 1/2 (b) 1 (c) 0 (d) $-1/2$

128. Let [.] denote the greatest integer function then the value of $\int_0^{1.5}x\left[x^2\right]dx$ is :. **[2011 RS]**

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{5}{4}$

129. The value of $\int_0^1\frac{8\log(1+x)}{1+x^2}dx$ is **[2011]**

(a) $\frac{\pi}{8}\log 2$ (b) $\frac{\pi}{2}\log 2$ (c) $\log 2$ (d) $\pi\log 2$

130. Let $p(x)$ be a function defined on **R** such that $p'(x)=p'(1-x)$, for all $x\in[0,1]$, $p(0)=1$ and $p(1)=41$.

Then $\int_0^1 p(x)dx$ equals **[2010]**

(a) 21 (b) 41 (c) 42 (d) $\sqrt{41}$

131. $\int_0^{\pi}[\cot x]dx$, where [.] denotes the greatest integer function, is equal to : **[2009]**

(a) 1 (b) -1 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

132. Let $I=\int_0^1\frac{\sin x}{\sqrt{x}}dx$ and $J=\int_0^1\frac{\cos x}{\sqrt{x}}dx$. Then which one of the following is true? **[2008]**

(a) $I>\frac{2}{3}$ and $J>2$ (b) $I<\frac{2}{3}$ and $J<2$

(c) $I<\frac{2}{3}$ and $J>2$ (d) $I>\frac{2}{3}$ and $J<2$

133. The solution for x of the equation

$\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2-1}}=\frac{\pi}{2}$ is **[2007]**

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$

(c) 2 (d) None of these

134. Let $F(x)=f(x)+f\left(\frac{1}{x}\right)$, where $f(x)=\int_l^x\frac{\log t}{1+t}dt$, Then $F(e)$ equals **[2007]**

(a) 1 (b) 2 (c) 1/2 (d) 0,

135. The value of $\int_1^a[x]f'(x)dx$, $a>1$ where $[x]$ denotes the greatest integer not exceeding x is **[2006]**

(a) $af(a)-\{f(1)+f(2)+.............f([a])\}$

(b) $[a]f(a)-\{f(1)+f(2)+.............f([a])\}$

(c) $[a]f([a])-\{f(1)+f(2)+.............f(a)\}$

(d) $af([a])-\{f(1)+f(2)+.............f(a)\}$

136. $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}}[(x+\pi)^3+\cos^2(x+3\pi)]dx$ is equal to **[2006]**

(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32}+\frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}-1$

137. $\int_0^{\pi}xf(\sin x)dx$ is equal to **[2006]**

(a) $\pi\int_0^{\pi}f(\cos x)dx$ (b) $\pi\int_0^{\pi}f(\sin x)dx$

(c) $\frac{\pi}{2}\int_0^{\pi/2}f(\sin x)dx$ (d) $\pi\int_0^{\pi/2}f(\cos x)dx$

138. The value of integral, : $\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is **[2006]**

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) 1

139. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, a > 0, is **[2005]**

(a) $a\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{a}$ (d) 2π

140. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then **[2005]**

(a) $I_2 > I_1$ (b) $I_1 > I_2$ (c) $I_3 = I_4$ (d) $I_3 > I_4$

141. Let $f : R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals **[2005]**

(a) 24 (b) 36 (c) 12 (d) 18

142. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$

and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$,

then the value of $\frac{I_2}{I_1}$ is **[2004]**

(a) 1 (b) –3 (c) –1 (d) 2

143. If $\int_0^{\pi} xf(\sin x)dx = A \int_0^{\pi/2} f(\sin x)dx$, then A is **[2004]**

(a) 2π (b) π (c) $\frac{\pi}{4}$ (d) 0

144. The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}} dx$ is **[2004]**

(a) 3 (b) 1 (c) 2 (d) 0

145. The value of $\int_{-2}^{3} |1-x^2| dx$ is **[2004]**

(a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$

146. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is **[2003]**

(a) $\frac{1}{n+1} + \frac{1}{n+2}$ (b) $\frac{1}{n+1}$

(c) $\frac{1}{n+2}$ (d) $\frac{1}{n+1} - \frac{1}{n+2}$.

147. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$, is **[2003]**

(a) $e + \frac{e^2}{2} + \frac{5}{2}$ (b) $e - \frac{e^2}{2} - \frac{5}{2}$

(c) $e + \frac{e^2}{2} - \frac{3}{2}$ (d) $e - \frac{e^2}{2} - \frac{3}{2}$.

148. If $f(a+b-x) = f(x)$ then $\int_a^b xf(x)dx$ is equal to **[2003]**

(a) $\frac{a+b}{2}\int_a^b f(a+b+x)dx$ (b) $\frac{a+b}{2}\int_a^b f(b-x)dx$

(c) $\frac{a+b}{2}\int_a^b f(x)dx$ (d) $\frac{b-a}{2}\int_a^b f(x)dx$.

149. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \sec^2 t\,dt}{x \sin x}$ is **[2003]**

(a) 0 (b) 3 (c) 2 (d) 1

150. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and

$F(t) = \int_0^t f(t-y)g(y)\,dy$, then **[2003]**

(a) $F(t) = te^{-t}$ (b) $F(t) = 1 - te^{-t}(1+t)$

(c) $F(t) = e^t - (1+t)$ (d) $F(t) = te^t$.

151. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is **[2002]**

(a) $\frac{\pi^2}{4}$ (b) π^2 (c) zero (d) $\frac{\pi}{2}$

152. $\int_0^2 [x^2]dx$ is **[2002]**

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$

(c) $\sqrt{2}-1$ (d) $-\sqrt{2}-\sqrt{3}+5$

153. $I_n = \int_0^{\pi/4} \tan^n x\,dx$ then $\lim_{n \to \infty} n[I_n + I_{n+2}]$ equals **[2002]**

(a) $\frac{1}{2}$ (b) 1 (c) ∞ (d) zero

154. $\int_0^{10\pi} |\sin x|\,dx$ is **[2002]**

(a) 20 (b) 8 (c) 10 (d) 18

TOPIC 4 Reduction Formulae for Definite Integration, Gamma & Beta Function, Walli's Formula, Summation of Series by Integration

155. Let a function $f:[0,5]\to \mathbf{R}$ be continuous, $f(1)=3$ and F be defined as:

$$F(x)=\int_1^x t^2 g(t)dt,\ \text{where } g(t)=\int_1^t f(u)du$$

Then for the function F, the point $x=1$ is : **[Jan. 9, 2020 (II)]**

(a) a point of local minima.
(b) not a critical point.
(c) a point of local maxima.
(d) a point of inflection.

156. $\lim\limits_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+.....+\frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to : **[April 10, 2019 (I)]**

(a) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$ (b) $\frac{4}{3}(2)^{4/3}$

(c) $\frac{3}{2}(2)^{4/3}-\frac{4}{3}$ (d) $\frac{4}{3}(2)^{3/4}$

157. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+.....+\frac{1}{5n}\right)$ is equal to : **[Jan. 12, 2019 (II)]**

(a) $\frac{\pi}{4}$ (b) $\tan^{-1}(3)$ (c) $\frac{\pi}{2}$ (d) $\tan^{-1}(2)$

158. If $\lim\limits_{n\to\infty}\frac{1^a+2^a+.........+n^a}{(n+1)^{a-1}[(na+2)+.....(na+n)]}=\frac{1}{60}$

for some positive real number a, then a is equal to : **[Online April 9, 2017]**

(a) 7 (b) 8 (c) $\frac{15}{2}$ (d) $\frac{17}{2}$

159. $\lim\limits_{n\to\infty}\left(\frac{(n+1)(n+2)...3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to: **[2016]**

(a) $\frac{9}{e^2}$ (b) $3\log 3-2$

(c) $\frac{18}{e^4}$ (d) $\frac{27}{e^2}$

160. $f(x)=\int\frac{dx}{\sin^6 x}$ is a polynomial of degree **[Online May 26, 2012]**

(a) 5 in $\cot x$ (b) 5 in $\tan x$
(c) 3 in $\tan x$ (d) 3 in $\cot x$

161. $\lim\limits_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}...........+\frac{1}{n}\sec^2 1\right]$ equals **[2005]**

(a) $\frac{1}{2}\sec 1$ (b) $\frac{1}{2}$ cosec 1

(c) tan 1 (d) $\frac{1}{2}\tan 1$

162. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}}$ is **[2004]**

(a) $e+1$ (b) $e-1$ (c) $1-e$ (d) e

163. $\lim\limits_{n\to\infty}\frac{1+2^4+3^4+...n^4}{n^5}-\lim\limits_{n\to\infty}\frac{1+2^3+3^3+...n^3}{n^5}$ **[2003]**

(a) $\frac{1}{5}$ (b) $\frac{1}{30}$ (c) Zero (d) $\frac{1}{4}$

164. $\lim\limits_{n\to\infty}\frac{1^p+2^p+3^p+.....+n^p}{n^{p+1}}$ is **[2002]**

(a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$

(c) $\frac{1}{p}-\frac{1}{p-1}$ (d) $\frac{1}{p+2}$

Hints & Solutions

1. (Bonus)

$$\lim_{x\to 1}\frac{\frac{1}{2}\sin(x-1)^4}{(x-1)\sin(x-1)}$$

Let $x-1=h$ when $x\to 1$ then $h\to 0$

$$\lim_{h\to 0}\frac{\sin h^4}{h^4}\times\frac{h}{\sin h}\times h^2=1\times 1\times 0=0$$

(No any option is correct)

2. (d) $\int (e^{2x}+2e^x-e^{-x}-1)\cdot e^{(e^x+e^{-x})}dx$

$$I=\int (e^{2x}+e^x-1)\cdot e^{(e^x+e^{-x})}dx+\int (e^x-e^{-x})e^{(e^x+e^{-x})}dx$$

$$=\int e^x(e^x+1-e^{-x})\cdot e^{(e^x+e^{-x})}dx+e^{(e^x+e^{-x})}$$

$$=\int (e^x-e^{-x}+1)e^{(e^x+e^{-x}+x)}dx+e^{(e^x+e^{-x})}$$

Let $e^x+e^{-x}+x=t\Rightarrow (e^x+e^{-x}+1)dx=dt$

$$=\int e^t\,dt+e^{(e^x+e^{-x})}=e^t+e^{(e^x+e^{-x})}+C$$

$$=e^{(e^x+e^{-x}+x)}+e^{(e^x+e^{-x})}+C$$

$$=(e^x+1).e^{(e^x+e^{-x})}+C$$

So, $g(x)=1+e^x$ and $g(0)=2$

3. (d) Let $\sin\theta=t\Rightarrow\cos\theta\,d\theta=dt$

$$\int\frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta=\frac{dt}{5+7t-2+2t^2}$$

$$\Rightarrow\frac{1}{2}\int\frac{dt}{\left(t+\frac{7}{4}\right)^2-\left(\frac{5}{4}\right)^2}=\frac{1}{5}\ln\left|\frac{t+\frac{1}{2}}{t+3}\right|+C$$

$$=\frac{1}{5}\ln\left|\frac{2t+1}{t+3}\right|+C=\frac{1}{5}\ln\left|\frac{2\sin\theta+1}{\sin\theta+3}\right|+C$$

$$\therefore B(\theta)=\frac{2\sin\theta+1}{2(\sin\theta+3)}\text{ and }A=\frac{1}{5}$$

$$\Rightarrow\frac{B(\theta)}{A}=\frac{5(2\sin\theta+1)}{(\sin\theta+3)}$$

4. (a) $\int\frac{x^2}{(x\sin x+\cos x)^2}dx$

$$\because\frac{d}{dx}(x\sin x+\cos x)=x\cos x$$

$$=\int\underset{\text{II}}{\frac{x\cos x}{(x\sin x+\cos x)^2}}\underset{\text{I}}{\left(\frac{x}{\cos x}\right)}dx$$

$$=\frac{x}{\cos x}\left[\frac{-1}{x\sin x+\cos x}\right]-\int\frac{x\sin x+\cos x}{\cos^2 x}\left[\frac{-1}{x\sin x+\cos x}\right]dx$$

$$=\frac{x}{\cos x}\left[\frac{-1}{x\sin x+\cos x}\right]+\int\sec^2 x\,dx$$

$$=\frac{-x\sec x}{x\sin x+\cos x}+\tan x+C$$

5. (d) $\int\frac{\sqrt{x}}{(1+x)^2}dx\ (x>0)$

Put $x=\tan^2\theta\Rightarrow 2x\,dx=2\tan\theta\sec^2\theta\,d\theta$

$$I=\int\frac{2\tan^2\theta\cdot\sec^2\theta}{\sec^4\theta}d\theta=\int 2\sin^2\theta d\theta$$

$$=\theta-\frac{\sin 2\theta}{2}+C$$

$$\Rightarrow f(x)=\theta-\frac{1}{2}\times\frac{2\tan\theta}{1+\tan^2\theta}+C$$

$$f(x)=\theta-\frac{\tan\theta}{1+\tan^2\theta}+C=\tan^{-1}\sqrt{x}-\frac{\sqrt{x}}{1+x}+C$$

Now $f(3)-f(1)=\tan^{-1}(\sqrt{3})-\frac{\sqrt{3}}{1+3}-\tan^{-1}(1)+\frac{1}{2}$

$$=\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$$

6. (a) $I=\int\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)dx=\int\underset{\text{I}}{\tan^{-1}\sqrt{x}}\cdot\underset{\text{II}}{1}\,dx$

$$= x\tan^{-1}\sqrt{x} - \int \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}.x\ dx + C$$

$$= x\tan^{-1}\sqrt{x} - \frac{1}{2}\int\frac{t\cdot 2t\,dt}{1+t^2} + C$$

(Put $x = t^2 \Rightarrow dx = 2t\,dt$)

$$= x\tan^{-1}\sqrt{x} - \int\frac{t^2}{1+t^2}dt + C$$

$$= x\tan^{-1}\sqrt{x} - t + \tan^{-1}t + C$$

$$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$$

$$\Rightarrow A(x) = x+1 \Rightarrow B(x) = -\sqrt{x}$$

7. **(a)** $I = \int\frac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$

$$= \int\left(\frac{x-3}{x+4}\right)^{\frac{-6}{7}}\frac{1}{(x+4)^2}dx$$

Let $\frac{x-3}{x+4} = t^7$,

Differentiate on both sides, we get

$$\frac{7}{(x+4)^2}dx = 7t^6 dt$$

Hence, $I = \int t^{-6}t^6 dt = t + C = \left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$

8. **(c)** $I = \int\frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)}$

$$= \int\frac{\sec^2\theta}{\frac{1+\tan^2\theta}{1-\tan^2\theta} + \frac{2\tan\theta}{1-\tan^2\theta}}d\theta$$

$$= \int\frac{\sec^2\theta(1-\tan^2\theta)}{(1+\tan\theta)^2}d\theta$$

$$= \int\frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta}d\theta$$

Let $\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$, then

$$I = \int\left(\frac{1-t}{1+t}\right)dt = \int\left(-1+\frac{2}{1+t}\right)dt$$

$= -t + 2\log(1+t) + C$

$= -\tan\theta + 2\log(1+\tan\theta) + C$

Hence, by comparison $\lambda = -1$ and $f(x) = 1 + \tan\theta$

9. **(d)** Let $I = \int\frac{\cos x\,dx}{\sin^3 x(1+\sin^6 x)^{2/3}}$

$= f(x)(1+\sin^6 x)^{1/\lambda} + c$ ...(i)

If $\sin x = t$

then, $\cos x\,dx = dt$

$$I = \int\frac{dt}{t^3\left(1+t^6\right)^{\frac{2}{3}}} = \int\frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

Put $1+\frac{1}{t^6} = r^3 \Rightarrow \frac{dt}{t^7} = \frac{-1}{2}r^2 dr$

$$-\frac{1}{2}\int\frac{r^2 dr}{r^2} = -\frac{1}{2}r + c$$

$$= -\frac{1}{2}\left(\frac{\sin^6 x+1}{\sin^6 x}\right)^{\frac{1}{3}} + c = -\frac{1}{2\sin^2 x}(1+\sin^6 x)^{\frac{1}{3}} + c$$

$f(x) = -\frac{1}{2}\operatorname{cosec}^2 x$ and $\lambda = 3$ [from eqn. (i)]

$$\therefore\ \lambda f\left(\frac{\pi}{3}\right) = -2$$

10. **(c)** Given integral, $I = \int\frac{(2x^3-1)dx}{x^4+x} = \int\frac{(2x-x^{-2})dx}{x^2+x^{-1}}$

Put $x^2 + x^{-1} = u \Rightarrow (2x - x^{-2})dx = du$

$$\Rightarrow I = \int\frac{du}{u} = \log|u| + c = \log|x^2 + x^{-1}| + c$$

$$= \log\left|\frac{x^3+1}{x}\right| + c$$

11. **(b)** Given integral

$$\int\frac{\tan x + \tan\alpha}{\tan x - \tan\alpha}dx = \int\frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx$$

Let $x - \alpha = t \Rightarrow dx = dt$

$$= \int\frac{\sin(t+2\alpha)}{\sin t}dt = \int[\cos 2\alpha + \sin 2\alpha.\cot t]dt$$

$= \cos 2\alpha\,.\,t + \sin 2\alpha\,.\,\log|\sin t| + c$

$= (x-\alpha)\,.\,\cos 2\alpha + \sin 2\alpha\,.\,\log|\sin(x-\alpha)| + c$

12. **(a)** Let $I = \int\frac{dx}{(x^2-2x+10)^2} = \int\frac{dx}{((x-1)^2+9)^2}$

Let $(x-1)^2 = 9\tan^2\theta$ ...(i)

$$\Rightarrow \tan\theta = \frac{x-1}{3}$$

After differentiating equation ...(i), we get
$2(x-1)\,dx = 18\tan\theta\sec^2\theta\,d\theta$

$$\therefore\ I = \int \frac{18\tan\theta\sec^2\theta d\theta}{2\times 3\tan\theta\times 81\sec^4\theta}$$

$$I = \frac{1}{27}\int\cos^2\theta d\theta = \frac{1}{27}\times\frac{1}{2}\int(1+\cos 2\theta)d\theta$$

$$I = \frac{1}{54}\left\{\theta+\frac{\sin 2\theta}{2}\right\}+c$$

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{1}{2}\times\frac{2\left(\frac{x-1}{3}\right)}{1+\left(\frac{x-1}{3}\right)^2}\right]+c$$

$$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{3(x-1)}{x^2-2x+10}\right]+c$$

Compare it with A $\left[\tan^{-1}\left(\frac{x-1}{b}\right)+\frac{f(x)}{x^2-2x+10}\right]+c$,

we get: $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

13. **(d)** Since, function f(x) have local extreem points at $x = -1, 0, 1$. Then
$f(x) = K(x+1)x(x-1)$
$= K(x^3 - x)$

$$\Rightarrow f(x) = K\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+C \quad \text{(using integration)}$$

$\Rightarrow f(0) = C$

$$\because f(x) = f(0) \Rightarrow K\left(\frac{x^4}{4}-\frac{x^2}{2}\right) = 0$$

$$\Rightarrow \frac{x^2}{2}\left(\frac{x^2}{2}-1\right) = 0 \quad \Rightarrow x = 0, 0, \sqrt{2}, -\sqrt{2}$$

$$\therefore S = \{0, -\sqrt{2}, \sqrt{2}\}$$

14. **(a)** $I = \int \sec^{\frac{2}{3}} x . \text{cosec}^{\frac{4}{3}} dx$

$$I = \int \frac{\sec^2 x dx}{\tan^{\frac{4}{3}} x}$$

Put $\tan x = z$
$\Rightarrow \sec^2 x\, dx = dz$

$$\Rightarrow I = \int z^{-\frac{4}{3}}\cdot dz = \frac{z^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+C \Rightarrow I = -3(\tan x)^{\frac{-1}{3}}+C$$

15. **(a)** Given,

$$\int e^{\sec x}\left(\sec x\tan x\, f(x)+\left(\sec x+\tan x+\sec^2 x\right)\right)dx$$
$$= e^{\sec x} f(x)+C \quad ...(i)$$

$$\because \int e^{g(x)}\left((g'(x)f(x))+f'(x)\right)dx = e^{g(x)}\times f(x)+C$$

Our comparing above equation by equation (i),

$$f(x) = \int\left((\sec x\tan x)+\sec^2 x\right)dx$$

$\therefore f(x) = \sec x + \tan x + C$

16. **(c)** $$\int\frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)}dx = \int\frac{2\cos\frac{x}{2}.\sin\frac{5x}{2}}{2\cos\frac{x}{2}.\sin\frac{x}{2}}dx$$

$$= \int\frac{\sin 3x+\sin 2x}{\sin x}dx$$

$$= \int(3-4\sin^2 x+2\cos x)dx$$

[$\because \sin 2x = 2\sin x\cos x$ and $\sin 3x = 3\sin x - 4\sin^3 x$]

$$= \int(3-2(1-\cos 2x)+2\cos x)dx$$

$$= \int(1+2\cos x+2\cos 2x)dx$$

$= x + 2\sin x + \sin 2x + c$

17. **(d)** Let, $\int\frac{dx}{x^3(1+x^6)^{\frac{2}{3}}} = \int\frac{dx}{x^7(1+x^{-6})^{\frac{2}{3}}}$

Put $1 + x^{-6} = t^3 \Rightarrow -6^{-7}dx = 3t^2 dt \Rightarrow \frac{dx}{x^7} = \left(-\frac{1}{2}\right)t^2 dt$

Now, $I = \int\left(-\frac{1}{2}\right)\frac{t^2 dt}{t^2} = -\frac{1}{2}t+C$

$$= -\frac{1}{2}(1+x^{-6})^{\frac{1}{3}}+C = -\frac{1}{2}\frac{(1+x^6)^{\frac{1}{3}}}{x^2}+C$$

$$= -\frac{1}{2x^3}x(1+x^6)^{\frac{1}{3}}+C$$

Hence, $f(x) = -\frac{1}{2x^3}$

18. **(c)** Let the integral, $I=\int \cos(\ln x)\,dx$

$$\Rightarrow I=\cos(\ln x).x-\int \frac{-\sin(\ln x)}{x}.x\,dx$$

$$=x\cos(\ln x)+\int \sin(\ln x)dx$$

$$=x\cos(\ln x)+\sin(\ln x).x-\int \frac{\cos(\ln x)}{x}.x\,dx$$

$$=x\cos(\ln x)+\sin(\ln x).x-I$$

$$\Rightarrow 2I=x(\cos(\ln x)+\sin(\ln x))+C$$

$$\Rightarrow I=\frac{x}{2}[\cos(\ln x)+\sin(\ln x)]+C$$

19. **(b)** $I=\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx=\int \frac{3x^{13}+2x^{11}}{x^{16}\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}dx$

$$I=\int \frac{\frac{3}{x^3}+\frac{2}{x^5}}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}dx$$

Let $2+\frac{3}{x^2}+\frac{1}{x^4}=t,\ -2\left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx=dt$

Then, $I=\int \frac{-\frac{dt}{2}}{t^4}=-\frac{1}{2}\frac{t^{-4+1}}{-4+1}+C$

$$I=\frac{-1}{2}\times\frac{1}{(-3)}\frac{1}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^3}+C$$

$$I=\frac{1}{6}\frac{x^{12}}{(2x^4+3x^2+1)^3}+C$$

20. **(a)** $A(x)\left(\sqrt{1-x^2}\right)^m+C=\int \frac{\sqrt{1-x^2}}{x^4}dx$

$$=\int \frac{\sqrt{\frac{1}{x^2}-1}}{x^3}dx$$

Let $\frac{1}{x^2}-1=u^2$

$$\Rightarrow -\frac{2}{x^3}=\frac{2u\,du}{dx}$$

$$\frac{dx}{x^3}=-u\,du$$

$$A(x)\left(\sqrt{1-x^2}\right)^m+C=\int(-u^2)du=-\frac{u^3}{3}+C$$

$$=-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}+C$$

$$=-\frac{1}{3}\cdot\frac{1}{x^3}\cdot(1-x^2)^{\frac{3}{2}}+C$$

$$=\frac{-1}{3x^3}\left(\sqrt{1-x^2}\right)^3+C$$

Compare both sides,

$$\Rightarrow A(x)=-\frac{1}{3x^3}\text{ and } m=3$$

$$\Rightarrow (A(x))^3=\frac{-1}{27x^9}$$

21. **(c)** Let $I=\int \frac{x+1}{\sqrt{2x-1}}dx$

Put $\sqrt{2x-1}=t$

$\therefore\ 2x-1=t^2\Rightarrow dx=t\,dt$

$$I=\int \frac{(t^2+3)}{2}dt=\frac{t^3}{6}+\frac{3t}{2}+C$$

$$=\frac{(2x-1)^{\frac{3}{2}}}{6}+\frac{3}{2}(2x-1)^{\frac{1}{2}}+C$$

$$=\sqrt{2x-1}\left(\frac{x+4}{3}\right)+C$$

$$=f(x).\sqrt{2x-1}+C$$

Hence, $f(x)=\frac{x+4}{3}$

22. **(a)** Let, $I=\int \frac{(\sin^n\theta-\sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$

Let $\sin\theta=u$

$\Rightarrow \cos\theta\,d\theta=du$

$$\therefore\ I=\int \frac{(u^n-u)^{\frac{1}{n}}}{u^{n+1}}du$$

$$=\int \frac{\left(1-\frac{1}{u^{n-1}}\right)^{\frac{1}{n}}}{u^n}du=\int u^{-n}(1-u^{1-n})^{\frac{1}{n}}du$$

Let $1-u^{1-n}=v$

$\Rightarrow -(1-n)u^{-n}\,du=dv$

$$\Rightarrow u^{-n}\,du=\frac{dv}{n-1}$$

$$\therefore \quad I=\int v^{\frac{1}{n}} \cdot \frac{dv}{n-1}=\frac{1}{n-1} \cdot \frac{v^{\frac{1}{n}+1}}{\frac{1}{n}+1}$$

$$=\frac{n}{n^2-1} v^{\frac{n+1}{n}}+C=\frac{n}{n^2-1}\left(1-\frac{1}{u^{n-1}}\right)^{\frac{n+1}{n}}+C$$

$$=\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$$

23. (c, d) Consider the given integral

$$I=\int x\sqrt{\frac{2\sin(x^2-1)-2\sin(x^2-1)\cos(x^2-1)}{2\sin(x^2-1)+2\sin(x^2-1)\cos(x^2-1)}}dx$$

($\because$ sin 2 θ = 2sin θ cos θ)

$$\Rightarrow \quad I=\int x\sqrt{\frac{1-\cos(x^2-1)}{1+\cos(x^2-1)}}dx$$

$$\Rightarrow \quad I=\int x\left|\tan\left(\frac{x^2-1}{2}\right)\right| dx,$$

Now let $\frac{x^2-1}{2}=t \Rightarrow \frac{2x}{2}dx=dt$

$$\therefore \quad I=\int|\tan(t)|\,dt=\ln|\sec t|+C$$

or $\quad I=\ln\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c=\frac{1}{2}\ln\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

24. (a) $f(x)=\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx, x\geq 0$

$$=\int\frac{5x^8+7x^6}{x^{14}(x^{-5}+x^{-7}+2)^2}dx$$

$$=\int\frac{5x^{-6}+7x^{-8}}{(2+x^{-7}+x^{-5})^2}dx$$

Let $2+x^{-7}+x^{-5}=t$

$\Rightarrow \quad (-7x^{-8}-5x^{-6})dx=dt$

$$\Rightarrow \quad f(x)=\int\frac{-dt}{t^2}=\int -t^{-2}dt=t^{-1}+c$$

$$\Rightarrow \quad f(x)=\frac{1}{2+x^{-7}+x^{-5}}+c,\ f(0)=0 \Rightarrow c=0$$

$$\therefore \quad f(1)=\frac{1}{4}$$

25. (a) Let I

$$=\int\frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$$

$$=\int\frac{\sin^2 x\cos^2 x}{[(\sin^2 x+\cos^2 x)(\sin^3 x+\cos^3 x)]^2}dx$$

$$=\int\frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx=\int\frac{\tan^2 x\cdot\sec^2 x}{(1+\tan^3 x)^2}dx$$

Now, put $(1+\tan^3 x)=t$

$\Rightarrow$ $3\tan^2 x\sec^2 x\,dx=dt$

$$\therefore \quad I=\frac{1}{3}\int\frac{dt}{t^2}=-\frac{1}{3t}+C=\frac{-1}{3(1+\tan^3 x)}+C$$

26. (a) Let $I=\int\frac{\tan x}{1+\tan x+\tan^2 x}dx$

$$\Rightarrow I=\int\frac{\tan x+1+\tan^2 x}{\tan x+1+\tan^2 x}dx-\int\frac{(1+\tan^2 x)}{1+\tan x+\tan^2 x}$$

$$\Rightarrow I=x-\int\frac{\sec^2 x\,dx}{1+\tan x+\tan^2 x}$$

Put $\tan x=t \Rightarrow \sec^2 x\,.\,dx=dt$

$$\therefore I=x-\int\frac{dt}{t^2+t+\frac{1}{4}+1-\frac{1}{4}}$$

$$=x-\int\frac{dt}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow I=x-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+C$$

$$\Rightarrow I=x-\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x+1}{\sqrt{3}}\right)+C$$

$\therefore$ $A=3$ and $K=2$

27. (b) Suppose, $\frac{x-4}{x+2}=y \Rightarrow x-4=y(x+2)$

$$\Rightarrow x(1-y)=2y+4 \Rightarrow x=\frac{2y+4}{1-y}$$

So, $f(y)=2\left(\frac{2y+4}{1-y}\right)+1$

Now, $f(x)=2\left(\frac{2x+4}{1-x}\right)+1=\frac{3x+9}{1-x}$

$= \frac{3(x+3)}{1-x} = \frac{3(x-1+4)}{1-x} = -3 + \frac{12}{1-x}$

$\therefore \int f(x)\,dx = -12\log_e|1-x| - 3x + c$

28. **(a)** $\because 7-6x-x^2 = 16-(x+3)^2$

and $\frac{d}{dx}(7-6x-x^2) = -2x-6$

So, $\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = \int \frac{2x+6}{\sqrt{7-6x-x^2}}dx$

$-\int \frac{1}{\sqrt{16-(x+3)^2}}dx$

$= -2\sqrt{7-6x-x^2} - \sin^{-1}\left(\frac{x+3}{4}\right) + C$

Therefore, $A = -2$, & $B = -1$.

29. **(b)** $f\left(\frac{3x-4}{3x+4}\right) = x+2,\ x \neq -\frac{4}{3}$

Consider $\frac{3x-4}{3x+4} = t$

$\Rightarrow 3x-4 = 3tx+4t$

$\Rightarrow x = \frac{4t+4}{3-3t} + 2$

$\Rightarrow f(t) = \frac{10-2t}{3-3t}$

$\Rightarrow f(x) = \frac{2x-10}{3x-3}$

$\therefore \int f(x)dx = \int \frac{2x-10}{3x-3}dx$

$= \int \frac{2x}{3x-3}dx - 10\int \frac{dx}{3x-3}$

$= \frac{2}{3}\int \frac{x-1}{x-1}dx + \frac{2}{3}\int \frac{dx}{x-1} - \frac{10}{3}\int \frac{dx}{x-1}$

$= \frac{2x}{3} - \frac{8}{3}\ln(x-1) + C$

Here, $A = -\frac{8}{3}, B = \frac{2}{3}$

$\therefore (A, B) = \left(-\frac{8}{3}, \frac{2}{3}\right)$

30. **(a)** Let, $I = \int \sqrt{1+2\cot x \operatorname{cosec} x + 2\cot^2 x}\,.dx$

$\Rightarrow I = \int \sqrt{\frac{\sin^2 x + 2\cos x + 2\cos^2 x}{\sin^2 x}}\,.dx$

$\Rightarrow I = \int \sqrt{\frac{1+2\cos x + \cos^2 x}{\sin x}}\,.dx$

$\Rightarrow I = \int \left|\frac{1+\cos x}{\sin x}\right|.dx$

$\Rightarrow I = \int |\operatorname{cosec} x + \cot x|\,.dx$

$\Rightarrow I = \log|\operatorname{cosec} x - \cot x| + \log|\sin x| + C$

$\Rightarrow I = \log|1-\cos x| + C$

$\Rightarrow I = \log\left|2\sin^2\frac{x}{2}\right| + C$

$\Rightarrow I = \log\left|\sin^2\frac{x}{2}\right| + \log 2 + C$

$\Rightarrow I = 2\log\left|\sin\frac{x}{2}\right| + C_1$

31. **(a)** $\int \frac{dx}{\cos^3 x\sqrt{4\sin x\cos x}} = \int \frac{dx}{2\cos^4 x\sqrt{\tan x}}$

Let $\tan x = t^2 \Rightarrow \sec^2 x = 1 + t^4$

$\sec^2 x\, dx = 2t\, dt$

$= \int \frac{\sec^4 x\, dx}{2\sqrt{\tan x}} = \int \frac{\sec^2 x(\sec^2 x\, dx)}{2\sqrt{\tan x}}$

$= \int \frac{(1+t^4)2t\, dt}{2t} = \int (1+t^4)\, dt = t + \frac{t^5}{5} + k$

$= \sqrt{\tan x} + \frac{1}{5}\tan^{5/2} x + k\left[t = \sqrt{\tan x}\right]$

$A = \frac{1}{2}, B = \frac{5}{2}, C = \frac{1}{5}$

$A + B + C = \frac{16}{5}$

32. **(d)** Let $I = \int \frac{\log(t+\sqrt{1+t^2})}{\sqrt{1+t^2}}dt$

put $u = \log(t+\sqrt{1+t^2})$

$du = \frac{1}{t+\sqrt{1+t^2}}\cdot\left[\frac{t+\sqrt{1+t^2}}{\sqrt{1+t^2}}\right] = \frac{1}{\sqrt{1+t^2}}dt$

$\therefore I = \int u\, du = \frac{u^2}{2} + c$

Since, $I = \frac{1}{2}[g(t)]^2 + c$

$\therefore \quad g(t) = \log\ (t + \sqrt{1+t^2})$

Put t = 2

$g\ (b) = \log\ (2 + \sqrt{5})$

33. (d) Let $I = \int\left(1 + x - \frac{1}{x}\right)e^{x+1/x}dx$

$= \int e^{x+1/x}dx + \int\left(x - \frac{1}{x}\right)e^{x+1/x}dx$

$= x.e^{x+1/x} - \int x\left(1 - \frac{1}{x^2}\right)e^{x+1/x}dx + \int\left(x - \frac{1}{x}\right)e^{x+1/x}dx$

$= x.e^{x+\frac{1}{x}} - \int\left(x - \frac{1}{x}\right)e^{x+1/x}dx + \int\left(x - \frac{1}{x}\right)e^{x+1/x}dx$

$= xe^{x+\frac{1}{x}} + C$

34. (b) Let $I = \int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2}dx$

$I = \int\left(\frac{\sin x.\cos x}{\sin^3 x + \cos^3 x}\right)^2 dx$

$I = \int\left(\frac{\sin x.\cancel{\cos} x}{\cancel{\cos^3} x\,(1 + \tan^3 x)}\right)^2 dx$

$= \int\left(\frac{\sin x.\sec^2 x}{(1 + \tan^3 x)}\right)^2 dx$

Put $1 + \tan^3 x = t$

$dt = 3\tan^2 x \sec^2 x\, dx$ or $dx = \frac{dt}{3\tan^2 x \sec^2 x}$

$\therefore I = \int \frac{\sin^2 x.\sec^4 x}{t^2} \times \frac{dt}{3\tan^2 x \sec^2 x}$

$I = \frac{1}{3}\int \frac{\sin^2 x.\sec^4 x}{t^2} \times \frac{dt}{\frac{\sin^2 x}{\cos^2 x} \times \sec^2 x}$

$= \frac{1}{3}\int \frac{\cancel{\sin^2 x.\sec^4 x}}{t^2} \times \frac{dt}{\cancel{\sin^2 x \sec^4 x}}$

$\therefore I = \frac{1}{3}\int \frac{dt}{t^2} = \frac{1}{3}\int t^{-2}dt$

$I = \frac{1}{3}\left[\frac{t^{-2+1}}{-2+1}\right] + c = \frac{-1}{3}\left[\frac{1}{t}\right] + c$

or $I = -\frac{1}{3(1 + \tan^3 x)} + c$

35. (a) Let $I = \int x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

$\therefore I = 2\int \underset{II}{x}.\underset{I}{\tan^{-1}} x\, dx$

Applying Integration by parts

$I = 2\left[\tan^{-1} x \int x dx - \int\left(\frac{d}{dx}(\tan^{-1} x)\int x dx\right)dx\right]$

$I = 2\left[\frac{x^2}{2}\tan^{-1} x - \int \frac{1}{1+x^2} \times \frac{x^2}{2}dx\right] + c$

$I = 2\left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{x^2 + 1 - 1}{x^2 + 1}dx\right] + c$

$I = 2\left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{\cancel{x^2+1}}{\cancel{x^2+1}}dx + \frac{1}{2}\int \frac{1}{1+x^2}dx\right] + c$

$I = 2\left[\frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int 1.dx + \frac{1}{2}\tan^{-1} x\right] + c$

$I = 2\left[\frac{x^2}{2}\tan^{-1} x - \frac{x}{2} + \frac{1}{2}\tan^{-1} x\right] + c$

$I = x^2 \tan^{-1} x + \tan^{-1} x - x + c$

or $\boxed{I = -x + (x^2 + 1)\tan^{-1} x + c}$

36. (b) Let $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}dx$

$= \int \frac{(\sin^4 x)^2 - (\cos^4 x)^2}{1 - 2\sin^2 x \cos^2 x}dx$

$= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{1 - 2\sin^2 x \cos^2 x}dx$

$= \int \frac{[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x]\ [(\sin^2 x + \cos^2 x][\sin^2 x - \cos^2 x]}{1 - 2\sin^2 x \cos^2 x}dx$

$= -\int \cos 2x\, dx = \frac{-\sin 2x}{2} + c = -\frac{1}{2}\sin 2x + c$

37. (c) Let $\int f(x)dx = \psi(x)$

Let $I = \int x^5 f(x^3)dx$

put $x^3 = t$

$\Rightarrow 3x^2 dx = dt$

$I = \frac{1}{3}\int 3 \cdot x^2 \cdot x^3 \cdot f(x^3) \cdot dx$

$= \frac{1}{3}\int tf(t)dt = \frac{1}{3}\left[t\int f(t)dt - \int f(t)dt\right]$

$= \frac{1}{3}\left[t\psi(t) - \int \psi(t)dt\right]$

$= \frac{1}{3}\left[x^3\psi(x^3) - 3\int x^2\psi(x^3)dx\right] + c$

$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + c$

38. **(a)** Let $I = \int \frac{\cos 8x + 1}{\cot 2x - \tan 2x} dx$

Now, $D^r = \cot 2x - \tan 2x = \frac{\cos 2x}{\sin 2x} - \frac{\sin 2x}{\cos 2x}$

$= \frac{\cos^2 2x - \sin^2 2x}{\sin 2x \cos 2x} = \frac{2\cos 4x}{\sin 4x}$

$\therefore\ I = \int \frac{2\cos^2 4x}{\frac{2\cos 4x}{\sin 4x}} dx = \int \frac{2\cos^2 4x \,.\, \sin 4x}{2\cos 4x} dx$

$= \frac{1}{2}\int \sin 8x\, dx = -\frac{1}{2}\frac{\cos 8x}{8} + k = -\frac{1}{16}.\cos 8x + k$

Now, $-\frac{1}{16}.\cos 8x + k = A\cos 8x + k$

$\Rightarrow A = -\frac{1}{16}$

39. **(d)** $\int \frac{5\tan x}{\tan x - 2} dx = \int \frac{5\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} - 2} dx$

$= \int\left(\frac{5\sin x}{\cos x} \times \frac{\cos x}{\sin x - 2\cos x}\right)dx$

$= \int \frac{5\sin x\, dx}{\sin x - 2\cos x}$

$= \int\left(\frac{4\sin x + \sin x + 2\cos x - 2\cos x}{\sin x - 2\cos x}\right)dx$

$= \int \frac{(\sin x - 2\cos x) + (4\sin x + 2\cos x)}{\sin x - 2\cos x} dx$

$= \int \frac{(\sin x - 2\cos x) + 2(\cos x + 2\sin x)}{(\sin x - 2\cos x)} dx$

$= \int \frac{\sin x - 2\cos x}{\sin x - 2\cos x} dx + 2\int\left(\frac{\cos x + 2\sin x}{\sin x - 2\cos x}\right)dx$

$= \int dx + 2\int \frac{\cos x + 2\sin x}{\sin x - 2\cos x} dx = I_1 + I_2$

where, $I_1 = \int dx$ and $I_2 = 2\int \frac{\cos x + 2\sin x}{\sin x - 2\cos x} dx$

Let $\sin x - 2\cos x = t$

$\Rightarrow\ (\cos x + 2\sin x)\, dx = dt$

$\therefore\ I_2 = 2\int \frac{dt}{t} = 2\ln t + C = 2\, ln\,(\sin x – 2\cos x) + C$

Hence, $I_1 + I_2 = \int dx + 2\ln(\sin x - 2\cos x) + c$

$= x + 2ln\,|(\sin x - 2\cos x)| + k \Rightarrow a = 2$

40. **(a)** Let $f(x) = \int\left(\frac{x^2 + \sin^2 x}{1 + x^2}\right)\sec^2 x\, dx$

$= \int \frac{x^2\sec^2 x + \frac{\sin^2 x}{\cos^2 x}}{1 + x^2} dx$

$= \int \frac{x^2\sec^2 x + \tan^2 x}{1 + x^2} dx$

$= \int \frac{x^2(1 + \tan^2 x) + \tan^2 x}{1 + x^2} dx$

$= \int \frac{x^2 + \tan^2 x(1 + x^2)}{1 + x^2} dx$

$= \int \frac{x^2}{1 + x^2} dx + \int \tan^2 x\, dx$

$= \int \frac{x^2 + 1 - 1}{1 + x^2} dx + \int(\sec^2 x - 1)\, dx$

$= \int 1\, dx - \int \frac{dx}{1 + x^2} + \int sec^2 x dx - \int dx$

$= -\tan^{-1}x + \tan x + c$

Given : $f(0) = 0$

$\Rightarrow f(0) = -\tan^{-1}0 + \tan 0 + c \ \Rightarrow\ c = 0$

$\therefore f(x) = -\tan^{-1}x + \tan x$

Now, $f(1) = -\tan^{-1}(1) + \tan 1 = \tan 1 - \frac{\pi}{4}$

41. **(a)** Let $I = \int \frac{x^2 - x}{x^3 - x^2 + x - 1} dx$

$= \int \frac{x(x-1)}{x^2(x-1) + (x-1)} dx = \int \frac{x\, dx}{x^2 + 1} = \frac{1}{2}\int \frac{2x\, dx}{(x^2 + 1)}$

Let $x^2 + 1 = t \Rightarrow 2x\, dx = dt$

$\therefore\ I = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\log t + c$

$= \frac{1}{2}\log(x^2 + 1) + c$

where 'c' is the constant of integration.

42. **(d)** Statement - 2: $\cos^3 x$ is a periodic function.

It is a true statement.

Statement - 1

Given $f(x) = \int \cos^3 x\, dx = \int\left(\frac{\cos 3x}{4} + \frac{3\cos x}{4}\right)dx$

$= \frac{1}{4}\frac{\sin 3x}{3} + \frac{3}{4}\sin x = \frac{1}{12}\sin 3x + \frac{3}{4}\sin x$

Now, period of $\frac{1}{12}\sin 3x = \frac{2\pi}{3}$

Period of $\frac{3}{4}\sin x = 2\pi$

Hence period of $f(x) = \frac{\text{L.C.M.}(2\pi, 2\pi)}{\text{HCF of }(1,3)} = \frac{2\pi}{1} = 2\pi$

Thus, $f(x)$ is a periodic function of period 2π.
Hence, Statement - 1 is false.

43. **(c)** Let $I = \sqrt{2}\int \frac{\sin x dx}{\sin\left(x - \frac{\pi}{4}\right)}$

Let $x - \frac{\pi}{4} = t \Rightarrow dx = dt$

$\Rightarrow I = \sqrt{2}\int \frac{\sin\left(t + \frac{\pi}{4}\right)}{\sin t}dt = \frac{\sqrt{2}}{\sqrt{2}}\int\left(\frac{\sin t + \cos t}{\sin t}\right)dt$

$\Rightarrow I = \int(1 + \cot t)dt = t + \log|\sin t| + c_1$

$= x - \frac{\pi}{4} + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c_1$

$= x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c \left(\text{where } c = c_1 - \frac{\pi}{4}\right)$

44. **(c)** $I = \int \frac{dx}{\cos x + \sqrt{3}\sin x}$

$\Rightarrow I = \int \frac{dx}{2\left[\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right]}$

$= \frac{1}{2}\int \frac{dx}{\left[\sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x\right]} = \frac{1}{2}.\int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)}$

$\Rightarrow I = \frac{1}{2}.\int \text{cosec}\left(x + \frac{\pi}{6}\right)dx$

We know that

$\int \text{cosec}\, x\, dx = \log|(\tan x/2)| + C$

$\therefore I = \frac{1}{2}.\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

45. **(a)** $\int \frac{dx}{\cos x - \sin x} = \int \frac{dx}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}$

$= \int \frac{dx}{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}}\int \sec\left(x + \frac{\pi}{4}\right)dx$

$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2} + \frac{\pi}{8}\right)\right| + C$

$\left[\because \int \sec x\, dx = \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|\right]$

$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$

46. **(b)** $\int \frac{\sin x}{\sin(x - \alpha)}dx = \int \frac{\sin(x - \alpha + \alpha)}{\sin(x - \alpha)}dx$

$= \int \frac{\sin(x - \alpha)\cos\alpha + \cos(x - \alpha)\sin\alpha}{\sin(x - \alpha)}dx$

$= \int\{\cos\alpha + \sin\alpha\cot(x - \alpha)\}dx$

$= (\cos\alpha)x + (\sin\alpha)\log\sin(x - \alpha) + C$

Comparing with $Ax + B\log\sin(x - \alpha) + c$

$\therefore A = \cos\alpha, B = \sin\alpha$

47. **(d)** $\because f''(x) - g''(x) = 0$

Integrating, $f'(x) - g'(x) = c$;

$\Rightarrow f'(1) - g'(1) = c \Rightarrow 4 - 2 = c \Rightarrow c = 2.$

$\therefore f'(x) - g'(x) = 2$;

Integrating, $f(x) - g(x) = 2x + c_1$

$\Rightarrow f(2) - g(2) = 4 + c_1 \Rightarrow 9 - 3 = 4 + c_1$;

$\Rightarrow c_1 = 2 \therefore f(x) - g(x) = 2x + 2$

At $x = 3/2$, $f(x) - g(x) = 3 + 2 = 5$.

48. **(c)** $I = \int_1^2 e^x x^x(2 + \log_e x)dx$

$I = \int_1^2 e^x x^x[1 + (1 + \log_e x)]dx$

$= \int_1^2 e^x[x^x + x^x(1 + \log_e x)]dx$

$\because \int e^x(f(x) + f'(x))dx = e^x f(x) + c$

$\therefore I = \left[e^x x^x\right]_1^2$

$= e^2 \times 4 - e \times 1 = 4e^2 - e = e(4e - 1)$

49. **(a)** $\int_\alpha^{\alpha+1} \frac{dx}{(x + \alpha)(x + \alpha + 1)}$

$= \int_\alpha^{\alpha+1}\left[\frac{1}{x + \alpha} - \frac{1}{x + \alpha + 1}\right]dx$ [Using partial fraction]

$= \log\left(\frac{(x + \alpha)}{(x + \alpha + 1)}\right)\Bigg|_\alpha^{\alpha+1} = \log\left(\frac{2\alpha + 1}{2\alpha + 2}.\frac{2\alpha + 1}{2\alpha}\right)$

$= \log\frac{9}{8}$ (Given)

So, $\frac{(2\alpha+1)^2}{\alpha(\alpha+1)} = \frac{9}{2} \Rightarrow \quad 8\alpha^2 + 8\alpha + 2 = 9\alpha^2 + 9\alpha$

$\Rightarrow \alpha^2 + \alpha - 2 = 0 \Rightarrow \alpha = 1, -2$

50. **(c)** Let, $1 = \int x^2 . e^{-x^2} dx$

Put $-x^2 = t \Rightarrow -2x\, dx = dt$

$$1 = \int \frac{t^2 . e^t dt}{(-2)} = \frac{-1}{2} e^t (t^2 - 2t + 2)c$$

$$\therefore g(x) = \frac{-1}{2}(x^4 + 2x^2 + 2) \Rightarrow g(-1) = \frac{-5}{2}$$

51. **(b)** $I = \int x^5 e^{-4x^3} dx$

Put $-4x^3 = \theta$

$\Rightarrow -12x^2\, dx = d\theta$

$\Rightarrow x^2\, dx = -\frac{d\theta}{12}$

$$I = \int \frac{1}{48} \theta e^{\theta}\, d\theta = \frac{1}{48}[\theta\, e^{\theta} - e^{\theta}] + C$$

$$I = \frac{1}{48} e^{-4x^3} (-4x^3 - 1) + C$$

Then, by comparison

$f(x) = -4x^3 - 1$

52. **(c)** $I = \int \frac{dx}{(1+\sqrt{x}).\sqrt{x}\sqrt{1-x}}$

Put $1 + \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\Rightarrow I = \int \frac{2dt}{t\sqrt{2t - t^2}}$$

Again put $t = \frac{1}{z} \Rightarrow dt = \frac{-1}{z^2} dz$

$$\Rightarrow I = 2\int \frac{\frac{-1}{z^2} dz}{\frac{1}{z}\sqrt{\frac{2}{z} - \frac{1}{z^2}}} = 2\int \frac{-dz}{\sqrt{2z-1}}$$

$$= -2\sqrt{2z-1} + c = -2\sqrt{\frac{2}{t} - 1} + c$$

$$= -2\sqrt{\frac{2-t}{t}} + c = -2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + c$$

53. **(b)** $I = \int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^3(1+x^{-4})^{3/4}}$

Let $x^{-4} = y$

$\Rightarrow -4x^{-3}\, dx = dy \Rightarrow dx = \frac{-1}{4} x^3 dy$

$$\therefore I = \frac{-1}{4} \int \frac{x^3 dy}{x^3 (1+y)^{3/4}} = \frac{-1}{4} \int \frac{dy}{(1+y)^{3/4}}$$

$$= \frac{-1}{4} \times 4(1+y)^{1/4} = -(1+x^{-4})^{1/4} + C$$

$$= -\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$$

54. **(b)** $\int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}$

$$\int \frac{dx}{\left(\frac{x+1}{x-2}\right)^{3/4} (x-2)^2}, \quad \text{put } \frac{x+1}{x-2} = t$$

$$\frac{-3}{(x-2)^2} = \frac{dt}{dx}$$

$$\frac{dx}{(x-2)^2} = -\frac{dt}{3} = \frac{-1}{3}\int \frac{dt}{t^{3/4}} = -\frac{1}{3}\int t^{\frac{-3}{4}} dt$$

$$= \frac{1}{3}\left[\frac{t^{\frac{-3}{4}+1}}{\frac{-3}{4}+1}\right] = \frac{-4}{3}\left[\frac{x+1}{x-2}\right]^{1/4} + c$$

55. **(b)** $\int \frac{x^{5m-1} + 2x^{4m-1}}{(x^{2m} + x^m + 1)^3} dx = \int \frac{x^{5m-1} + 2x^{4m-1}}{x^{6m}(1 + x^{-m} + x^{-2m})^3} dx$

$$= \int \frac{x^{-m-1} + 2x^{-2m-1}}{(1 + x^{-m} + x^{-2m})^3} dx$$

Put $t = 1 + x^{-m} + x^{-2m}$

$$\therefore \frac{dt}{dx} = -mx^{-m-1} - 2mx^{-2m-1}$$

$$\Rightarrow \frac{dt}{-m} = (x^{-m-1} + 2x^{-2m-1}) dx$$

$$\therefore \int \frac{x^{5m-1} + 2x^{4m-1}}{(x^{2m} + x^m + 1)^3} dx = \frac{1}{-m}\int t^{-3} dt = \frac{1}{2mt^2} + C$$

$$= \frac{1}{2m(1 + x^{-m} + x^{-2m})^2} + C$$

$$= \frac{x^{4m}}{2m(x^{2m} + x^m + 1)^2} + C$$

$$\therefore f(x) = \frac{x^{4m}}{2m(x^{2m} + x^m + 1)^2}$$

56. **(b)** $I = \int \frac{x\, dx}{2 - x^2 + \sqrt{2 - x^2}}$

Put $t = \sqrt{2 - x^2}, \frac{dt}{dx} = \frac{1}{2\sqrt{2 - x^2}} . (-2x)$

$\Rightarrow \; -t\,dt = x\,dx$

$\therefore \quad I = \int \frac{(-t)\,dt}{t^2+t} = -\int \frac{1}{t+1}dt = -\log|t+1|$

$= -\log\left|\sqrt{2-x^2}+1\right| + c$

57. **(b)** Let $I = \int \frac{x^2-x+1}{x^2+1}.e^{\cot^{-1}x}\,dx$

Put $x = \cot t \Rightarrow -\text{cosec}^2 t\,dt = dx$
Now, $1+\cot^2 t = \text{cosec}^2 t$

$\therefore \quad I = \int \frac{e^t(\cot^2 t - \cot t + 1)}{(1+\cot^2 t)}(-\text{cosec}^2 t)\,dt$

$= -\int e^t(\text{cosec}^2 t - \cot t)\,dt$

$= \int e^t(\cot t - \text{cosec}^2 t)\,dt = e^t \cot t + C$

$= e^{\cot^{-1}x}(x) + C \equiv A(x).e^{\cot^{-1}x} + C$

$\Rightarrow A(x) = x$

58. **(a)** $\int \frac{x^6}{x+x^7}dx = \int \frac{x^6}{x(1+x^6)}dx = \int \frac{(1+x^6)-1}{x(1+x^6)}dx$

$= \int \frac{1}{x}dx - \int \frac{1}{x+x^7}dx = \ln|x| - p(x) + c$

59. **(d)** $\int \frac{(\log x-1)^2}{(1+(\log x)^2)^2}dx = \int \frac{1+(\log x)^2 - 2\log x}{\left[1+(\log x)^2\right]^2}dx$

$= \int \left[\frac{1}{(1+(\log x)^2)} - \frac{2\log x}{(1+(\log x)^2)^2}\right]dx$

$\therefore \quad I = \int \left[\frac{e^t}{1+t^2} - \frac{2t\,e^t}{(1+t^2)^2}\right]dt$

$= \int e^t\left[\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2}\right]dt$

$\left[\text{Which is of the form } \int e^x\left(f(x)+f'(x)\right)dx = f(x)\cdot e^x + c\right]$

$= \frac{e^t}{1+t^2} + c = \frac{x}{1+(\log x)^2} + c$

60. **(c)** $I_2 = \int_0^1 (1-x^{50})^{101}dx = \int_0^1 (1-x^{50})(1-x^{50})^{100}dx$

$I_2 = \int_0^1 (1-x^{50})^{100}dx - \int_0^1 \underbrace{x}_{I}\,\underbrace{x^{49}(1-x^{50})^{100}}_{II}dx$

$I_2 = I_1 + \left[\frac{x}{5050}(1-x^{50})^{101}\right]_0^1 - \int_0^1 \frac{(1-x^{50})^{101}}{5050}dx$

$I_2 = I_1 + 0 - \frac{I_2}{5050}$

$\Rightarrow \frac{5051}{5050}I_2 = I_1 \Rightarrow I_2 = \frac{5050}{5051}I_1$

$\Rightarrow \alpha = \frac{5050}{5051}$

61. **(c)** $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}dx$

$= \int_{-\pi/2}^{0} \frac{1}{1+e^{\sin x}}dx + \int_0^{\pi/2} \frac{1}{1+e^{\sin x}}dx$

$= \int_0^{\pi/2} \left(\frac{1}{1+e^{\sin x}} + \frac{1}{1+e^{-\sin x}}\right)dx$

$= \int_0^{\pi/2} \frac{1+e^{\sin x}}{1+e^{\sin x}}dx = \frac{\pi}{2}$

62. **(a)** $f(x) = |x-2| = \begin{cases} 2-x, & x<2 \\ x-2, & x\ge 2 \end{cases}$

$g(x) = f(f(x)) = \begin{cases} 2-f(x), & f(x)<2 \\ f(x)-2, & f(x)\ge 2 \end{cases}$

$= \begin{cases} 2-(2-x), & 2-x<2, & x<2 \\ (2-x)-2, & 2-x\ge 2, & x<2 \\ 2-(x-2), & x-2<2, & x\ge 2 \\ (x-2)-2, & x-2\ge 2, & x\ge 2 \end{cases}$

$= \begin{cases} -x & 0<x\le 0 \\ x & 0<x<2 \\ 4-x & 2\le x<4 \\ x-4 & x\ge 4 \end{cases}$

$\therefore \int_0^3 [g(x)-f(x)]dx$

$= \int_0^2 x\,dx + \int_2^3 (4-x)\,dx - \int_0^3 |x-2|\,dx = 1$

63. **(c)** $\int_{\pi/6}^{\pi/3}\left[\frac{1}{2}\frac{d(\tan^4 x)}{dx}\cdot\sin^4 3x + \frac{1}{2}\tan^4 x\cdot\frac{d(\sin^4 3x)}{dx}\right]dx$

$= \frac{1}{2}\int_{\pi/6}^{\pi/3} d(\tan^4 x\cdot\sin^4 3x)dx$

$= \left[\frac{\tan^4 x\sin^4 3x}{2}\right]_{\pi/6}^{\pi/3} = \frac{9\cdot 0}{2} - \frac{\frac{1}{9}\cdot 1}{2} = \frac{-1}{18}$

64. **(21)**

$\int_0^n \{x\}\,dx = n\int_0^1 x\cdot dx = \frac{n}{2}$

$$\Rightarrow \int_0^n [x]dx = \int_0^n (x-\{x\})dx = \frac{n^2}{2}-\frac{n}{2}$$

According to the questions,

$\frac{n}{2}, \frac{n^2-n}{2}, 10(n^2-n)$ are in GP

$$\therefore \left(\frac{n^2-n}{2}\right)^2 = \frac{n}{2}\times 10(n^2-n)$$

$$\Rightarrow n^2 = 21n \Rightarrow n = 21.$$

65. **(c)** $I = \int_{-\pi}^{\pi} |\pi - |x|| \, dx \quad [\because |\pi-|x|| \text{ is even}]$

$$= 2\int_0^{\pi} |\pi-|x|| \, dx$$

$$= 2\int_0^{\pi} (\pi - x)\,dx$$

$$= 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = 2\left(\pi^2 - \frac{\pi^2}{2}\right) = \pi^2.$$

66. **(a)** $\frac{k}{6} = \int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{3/2}} dx$

Let $x = \sin\theta;\ dx = \cos\theta\, d\theta$,

then $\int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{3/2}} dx = \int_0^{\frac{\pi}{6}} \frac{\sin^2\theta\cos\theta}{\cos^3\theta} d\theta$

$$\therefore \frac{k}{6} = \int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos^3\theta}\cdot\cos\theta\, d\theta$$

$$\Rightarrow \frac{k}{6} = \int_0^{\frac{\pi}{6}} \tan^2\theta\, d\theta = \int_0^{\frac{\pi}{6}} (\sec^2\theta - 1)\, d\theta$$

$$\Rightarrow \frac{k}{6} = (\tan\theta - \theta)_0^{\pi/6} = \left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right) = \frac{2\sqrt{3}-x}{6}$$

$$\Rightarrow k = 2\sqrt{3} - \pi$$

67. **(1.50)**

$$\int_0^2 ||x-1|-x|dx = \int_0^1 |1-x-x|dx + \int_1^2 |x-1-x|dx$$

$$= \int_0^1 (1-2x)dx + \int_{1/2}^1 (2x-1)dx + \int_1^2 dx$$

$$= [x-x^2]_0^{\frac{1}{2}} + [x^2-x]_{\frac{1}{2}}^1 + [x]_1^2$$

$$= \frac{1}{2}-\frac{1}{4}+(1-1)-\left(\frac{1}{4}-\frac{1}{2}\right)+2-1 = \frac{1}{4}+\frac{1}{4}+1 = \frac{3}{2}$$

68. **(1)**

$$\int_1^2 |2x-[3x]|\,dx$$

$$= \int_1^2 |3x-[3x]-x|\,dx$$

$$= \int_1^2 |\{3x\}-x|\,dx = \int_1^2 (x-\{3x\})dx$$

$$= \int_1^2 x dx - \int_1^2 \{3x\}\,dx$$

$$= \left[\frac{x^2}{2}\right]_1^2 - 3\int_0^{1/3} 3x\ dx$$

$$= \frac{(4-1)}{2} - 9\left[\frac{x^2}{2}\right]_0^{1/3} = \frac{3}{2}-\frac{1}{2} = 1$$

69. **(d)** $\int_0^1 (a+bx+cx^2)dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3}\Big|_0^1 = a + \frac{b}{2} + \frac{c}{3}$

Now, $f(1) = a+b+c, f(0) = a$ and $f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}$

Now, $\frac{1}{6}\left(f(1)+f(0)+4f\left(\frac{1}{2}\right)\right)$

$$= \frac{1}{6}\left(a+b+c+a+4\left(a+\frac{b}{2}+\frac{c}{4}\right)\right)$$

$$= \frac{1}{6}(6a+3b+2c) = a+\frac{b}{2}+\frac{c}{3}$$

Hence, $\int_0^1 f(x) = \frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

70. **(c)** $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x} dx$

$$= \int_0^{\pi}\left[\frac{x\sin^8 x}{\sin^8 x+\cos^8 x} + \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x}\right]dx$$

$$\left[\because \int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx\right]$$

$$= \int_0^{\pi} \frac{2\pi \sin^8 x}{\sin^8 x + \cos^8 x} dx$$

$$= 2\pi \int_0^{\pi/2} \left[\frac{\sin^8 x}{\sin^8 x + \cos^8 x} + \frac{\cos^8 x}{\sin^8 x + \cos^8 x} \right] dx$$

$$= 2\pi \int_0^{\pi/2} 1 dx = 2\pi \times \frac{\pi}{2} = \pi^2$$

71. **(b)** $f(x) = \frac{1}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$

$$f'(x) = \frac{-1}{2}\left(\frac{(6x^2 - 18x + 12)}{(2x^3 - 9x^2 - 12x + 4)^{3/2}} \right)$$

$$= \frac{-6(x-1)(x-2)}{2(2x^3 - 9x^2 + 12x + 4)^{3/2}}$$

$f(1) = \frac{1}{3}$ and $f(2) = \frac{1}{\sqrt{8}}$

It is increasing function

$$\frac{1}{3} < I < \frac{1}{\sqrt{8}}$$

$$\frac{1}{9} < I^2 < \frac{1}{8}$$

72. **(c)** $I = \frac{1}{(a+b)} \int_a^b x[f(x) + f(x+1)]dx$ ...(i)

$x \to a + b - x$

$$I = \frac{1}{(a+b)} \int_a^b (a+b-x)[f(a+b-x) + f(a+b+1-x)]dx$$

$$I = \frac{1}{(a+b)} \int_a^b (a+b-x)[f(x+1) + f(x)]dx \quad \text{...(ii)}$$

[$\because$ put $x \to x + 1$ in $f(a + b + 1 - x) = f(x)$]

Add (i) and (ii)

$$2I = \int_a^b [f(x+1) + f(x)]dx$$

$$2I = \int_a^b f(x+1)dx + \int_a^b f(x)dx$$

$$= \int_a^b f(a+b+1-x)dx + \int_a^b f(x)dx$$

$$2I = 2\int_a^b f(x)dx$$

$$\therefore \int_{a-1}^{b-1} f(x+1)dx \quad [\because \text{Put } x \to x+1]$$

73. **(a)** $4\alpha \left\{ \int_{-1}^{0} e^{\alpha x} dx + \int_0^2 e^{-\alpha x} dx \right\} = 5$

$$\Rightarrow 4\alpha \left\{ \frac{e^{\alpha x}}{\alpha}\Bigg|_{-1}^{0} + \frac{e^{-\alpha x}}{-\alpha}\Bigg|_0^2 \right\} = 5$$

$$\Rightarrow 4\alpha \left\{ \left(\frac{1 - e^{-\alpha}}{\alpha} \right) - \left(\frac{e^{-2\alpha} - 1}{\alpha} \right) \right\} = 5$$

$$\Rightarrow 4(2 - e^{-\alpha} - e^{-2\alpha}) = 5$$

Put $e^{-\alpha} = t$

$\Rightarrow 4t^2 + 4t - 3 = 0 \quad \Rightarrow (2t + 3)(2t - 1) = 0$

$\Rightarrow e^{-\alpha} = \frac{1}{2} \Rightarrow \alpha = \log_e 2$

74. **(a)** $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$

$$\frac{2\cos^2\theta}{\sin^2\theta} - \frac{5}{\sin\theta} + 4 = 0$$

$\Rightarrow 2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0, \sin\theta \neq 0$

$\Rightarrow 2\sin^2\theta - 5\sin\theta + 2 = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta - 2) = 0$

$\therefore \sin\theta = \frac{1}{2} \quad \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\therefore \int_{\pi/6}^{5\pi/6} \cos^2 3\theta \, d\theta = \int_{\pi/6}^{5\pi/6} \frac{1 + \cos 6\theta}{2} d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 6\theta}{6} \right]_{\pi/6}^{5\pi/6} = \frac{1}{2}\left[\frac{5\pi}{6} - \frac{\pi}{6} + \frac{1}{6}(0 - 0) \right]$$

$$= \frac{1}{2} \cdot \frac{4\pi}{6} = \frac{\pi}{3}$$

75. **(a)** Given, $\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$

Differentiating both sides,

$4(f(x))^3 . f'(x) = g'(x)(x-2) + g(x)$

Putting $x = 2$, $\frac{4(6)^3.1}{48} = g(2) \Rightarrow \lim_{x \to 2} g(x) = 18$

76. **(d)** $\int_0^{\pi/2} \frac{\cot x\, dx}{\cot x + \text{cosec}\, x}$

$= \int_0^{\pi/2} \frac{\cot x\, dx}{1+\cos x} = \int_0^{\pi/2} \left(1 - \frac{1}{1+\cos x}\right) dx$

$= [x]_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2\cos^2 \frac{x}{2}} dx = \frac{\pi}{2} - \frac{1}{2}\int_0^{\pi/2} \sec^2 \frac{x}{2} dx$

$= \frac{\pi}{2} - \left(\tan \frac{x}{2}\right)_0^{\pi/2} = \frac{\pi}{2} - [1] = \left(\frac{\pi}{2} - 1\right) = m\pi + mn$

$\therefore\ m = ,\ n = -2$, Hence, $mn = -1$

77. **(b)** $I = \int_0^{2\pi} [\sin 2x(1+\cos 3x)]dx$ ...(1)

$\because \int_0^a f(x) = \int_0^a f(a-x)dx$

$\therefore I = \int_0^{2\pi} [-\sin 2x(1+\cos 3x)]dx$ ...(2)

From (1) + (2), we get;

$2I = \int_0^{2\pi} (-1)dx \Rightarrow 2I = -(x)_0^{2\pi} \Rightarrow I = -\pi$

78. **(c)** Let, $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x . \text{cosec}^{\frac{4}{3}} x\, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1.dx}{\cos^{\frac{2}{3}} x . \sin^{\frac{4}{3}} x}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1dx}{\cos^2 x . \tan^{\frac{4}{3}} x} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 xdx}{\tan^{\frac{4}{3}} x}$

Let $\tan x = u$

$I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} u^{-\frac{4}{3}} du = \frac{3\left[u^{-\frac{1}{3}}\right]_{\frac{1}{\sqrt{3}}}}{-1}$

$= -3\left[3^{-\frac{1}{6}} - \frac{1}{3^{\frac{-1}{6}}}\right] = -3(3^{\frac{-1}{6}} - 3^{\frac{1}{6}})$

$= 3(3^{\frac{1}{6}} - 3^{\frac{1}{6}}) = (3^{\frac{7}{6}} - 3^{\frac{5}{6}})$

79. **(b)** Let $I = \int_0^{\pi/2} \frac{\sin^3 xdx}{\sin x + \cos x}$ ...(1)

Use the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2} \frac{\cos^3 x\, dx}{\sin x + \cos x}$ ...(2)

Adding equation (1) and (2), we get

$\Rightarrow\ 2I = \int_0^{\pi/2} \left(1 - \frac{1}{2}\sin(2x)\right) dx$

$\Rightarrow I = \frac{1}{2}\left[x + \frac{1}{4}\cos 2x\right]_0^{\pi/2}$

$\Rightarrow\ I = \frac{\pi - 1}{4}$

80. **(0)** $\int_0^1 x\cot^{-1}\left(1 - x^2 + x^4\right)dx = \int_0^1 x\tan^{-1}\left(\frac{1}{1+x^4-x^2}\right)$

$= \int_0^1 x\tan^{-1}\left(\frac{x^2 - \left(x^2-1\right)}{1 + x^2\left(x^2-1\right)}\right) dx$

$= \frac{1}{2}\int_0^1 1\tan^{-1} t^2 dt - \frac{1}{2}\int_{-1}^0 1\tan^{-1} k\ dk$

Put $x^2 = t \Rightarrow 2xdx = dt$ in the first integral

and $x^2 - 1 = k \Rightarrow 2xdx = dk$ in the second integral.

$= \frac{1}{2}\int_0^1 1\tan^{-1} tdt - \frac{1}{2}\int_0^1 1\tan^{-1} kdk$

$= \frac{1}{2}\left(t\tan^{-1} t\Big|_0^1 - \int_0^1 \frac{t}{1+t^2} dt\right)$

$-\frac{1}{2}\left(k\tan^{-1} k\Big|_0^1 - \int_{-1}^0 \frac{k}{1+k^2} dk\right)$

$= \frac{1}{2}\left(\frac{\pi}{4} - \left(\frac{1}{2}\ln\left(1+t^2\right)\Big|_0^1\right) - \frac{1}{2}\left(-\frac{\pi}{4} - \left(\frac{1}{2}\ln\left(1+k^2\right)\Big|_{-1}^0\right)\right)\right)$

$= \left(\frac{\pi}{8} - \frac{1}{4}\ln 2\right) - \left(\frac{-\pi}{8} - \frac{1}{4}10 - \ln 2\right) = \frac{\pi}{4} - \frac{1}{2}\ln 2$

81. **(d)** Using L' Hospital rule and Leibnitz theorem, we get

$$\lim_{x\to 2}\frac{\int_6^{f(x)} 2t\,dt}{(x-2)}=\lim_{x\to 2}\frac{2f(x)f'(x)-0}{1}$$

Putting $x=2, 2f(2)f'(2)=12f'(2)$ $\quad[\because f(2)=6]$

82. **(d)** $g(f(x))=\log\left(\frac{2-x\cos x}{2+x\cos x}\right), x>0$

Let $I=\int_{-\pi/4}^{\pi/4}\log\left(\frac{2-x\cos x}{2+x\cos x}\right)dx$ ...(i)

Use the property $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

Then, equation (i) becomes,

$I=\int_{-\pi/4}^{\pi/4}\log\left(\frac{2+x\cos x}{2-x\cos x}\right)dx$ ...(ii)

Adding (i) and (ii)

$$2I=\int_{-\pi/4}^{\pi/4}\log\left(\frac{2-x\cos x}{2+x\cos x}\cdot\frac{2+x\cos x}{2-x\cos x}\right)dx$$

$$2I=\int_{-\pi/2}^{\pi/2}\log(1)dx=0$$

$\Rightarrow I=0=\log 1$

83. **(a)** $f(x)=\int_0^x g(g)dt,$ ...(i)

$\because g$ is a non-zero even function.

$\therefore g(-x)=g(x),$ ...(ii)

Given, $f(x+5)=g(x)$ ...(iii)

From (i) $f'(x)=g(x)$

Let, $I=\int_0^x f(t)dt,$

Put $t=\lambda-5\Rightarrow I=\int_5^{x+5} f(\lambda-5)d\lambda$

$\because f(x+5)=g(x)$

$\Rightarrow f(-x+5)=g(-x)=g(x)$ ...(iv)

$I=\int_5^{x+5} f(\lambda-5)d\lambda$

$f(0)=0, g(x)$ is even $\Rightarrow f(x)$ is odd

$\therefore I=\int_5^{x+5}-f(5-\lambda)d\lambda$

$\Rightarrow I=\int_5^{x+5} g(\lambda)d\lambda=\int_{x+5}^{5} g(t)dt$ (from (iv))

84. **(c)** $f(x)=f(a-x)$

$g(x)+g(a-x)=4$

Let, the integral,

$I=\int_0^a f(x)g(x)dx$

$=\int_0^a f(a-x)\cdot g(a-x)dx$

$\left[\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$

$\Rightarrow I=\int_0^a f(x)[4-g(x)]dx$

$\Rightarrow I=\int_0^a 4f(x)dx-\int_0^a f(x)\cdot g(x)dx$

$\Rightarrow I=\int_0^a 4f(x)dx-I$

$\Rightarrow 2I=\int_0^a 4f(x)dx$

$\Rightarrow I=2\int_0^a f(x)dx$

85. **(d)** $I=\int_1^e\left\{\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^x\right\}\log_e x\,dx$

Let $\left(\frac{x}{e}\right)^x=t$

$\Rightarrow x\ln\left(\frac{x}{e}\right)=\ln t$

$\Rightarrow x(\ln x-1)=\ln t$

On differentiating both sides w.r. t*x* we get

$\ln x.dx=\frac{dt}{t}$

When $x=e$ then $t=1$ and when $x=1$ then $t=\frac{1}{e}$.

$$I=\int_{\frac{1}{e}}^1\left(t^2-\frac{1}{t}\right)\cdot\frac{dt}{t}=\int_{\frac{1}{e}}^1\left(t-\frac{1}{t^2}\right)dt$$

$$=\left(\frac{t^2}{2}+\frac{1}{t}\right)_{\frac{1}{e}}^1=\left(\frac{1}{2}+1\right)-\left(\frac{1}{2e^2}+e\right)=\frac{3}{2}-e-\frac{1}{2e^2}$$

86. **(a)** Let $f(x)=\dfrac{\sin^2 x}{\left[\dfrac{x}{\pi}\right]+\dfrac{1}{2}}$

So, $f(-x)=\dfrac{\sin^2(-x)}{\left[\dfrac{-x}{\pi}\right]+\dfrac{1}{2}}$ $\quad \because [-x]=-1-[x]$

$\Rightarrow f(-x)=\dfrac{\sin^2 x}{-1-\left[\dfrac{x}{\pi}\right]+\dfrac{1}{2}}=\dfrac{\sin^2 x}{-\dfrac{1}{2}-\left[\dfrac{x}{\pi}\right]}=-f(x)$

$\Rightarrow f(x)$ is odd function

Hence, $\int_{-2}^{2} f(x)\,dx=0$

87. **(b)** $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\dfrac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$

$=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\dfrac{\tan^5 x.\sec^2 x}{2\dfrac{\sin x}{\cos x}\left(\left(\tan^5 x\right)^2+1\right)}$

$=\dfrac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\dfrac{\tan^4 x.\sec^2 x}{\left(\tan^5 x\right)^2+1}dx.$

Let $\tan^4 x=t.$

$5\tan^4 x\,.\,\sec^2 x\,dx=dt.$

When $x\to\dfrac{\pi}{4}$ then $t\to 1$

and $x\to\dfrac{\pi}{6}$ then $t\to\left(\dfrac{1}{\sqrt{3}}\right)^5$

$\therefore\ I=\dfrac{1}{10}\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1}\dfrac{dt}{t^2+1}$

$=\dfrac{1}{10}\left(\dfrac{\pi}{4}-\tan^{-1}\left(\dfrac{1}{9\sqrt{3}}\right)\right)$

88. **(d)** $I=\int_a^b (x^4-2x^2)\,dx$

$\Rightarrow \dfrac{dI}{dx}=x^4-2x^2=0$ (for minimum)

$\Rightarrow x=0,\pm\sqrt{2}$

Also, $I=\left[\dfrac{x^5}{5}-\dfrac{2x^3}{3}\right]_a^b$

For $a=0$, $b=\sqrt{2}$

$I=\dfrac{-8\sqrt{2}}{15}$

For $a=-\sqrt{2}$, $b=0$

$I=\dfrac{-8\sqrt{2}}{15}.$

For $a=\sqrt{2}$, $b=-\sqrt{2}$

$I=\dfrac{16\sqrt{2}}{15}.$

For $a=-\sqrt{2}$, $b=\sqrt{2}$

$I=\dfrac{-16\sqrt{2}}{15}.$

$\therefore$ I is minimum when $(a,b)=\left(-\sqrt{2},\sqrt{2}\right)$

89. **(a)** $\int_0^x f(t)dt=x^2+\int_x^1 t^2 f(t)dt$

$\Rightarrow f(x)=2x-x^2 f(x)$

$\Rightarrow f(x)=\dfrac{2x}{1+x^2}$

$\Rightarrow f'(x)=\dfrac{2(1-x^2)}{(1+x^2)^2}$

Then,

$f'(1/2)=\dfrac{2\left(1-\dfrac{1}{4}\right)}{\left(1+\dfrac{1}{4}\right)^2}=\dfrac{3}{2}\times\dfrac{16}{25}=\dfrac{24}{25}$

90 **(c)** $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\dfrac{dx}{[x]+[\sin x]+4}$

$=\int_{-\frac{\pi}{2}}^{-1}\dfrac{dx}{-2-1+4}+\int_{-1}^{0}\dfrac{dx}{-1-1+4}+\int_{0}^{1}\dfrac{dx}{0+0+4}+\int_{1}^{\frac{\pi}{2}}\dfrac{dx}{1+0+4}$

$=\left(-1+\dfrac{\pi}{2}\right)+\dfrac{1}{2}(0+1)+\dfrac{1}{4}(1-0)+\dfrac{1}{5}\left(\dfrac{\pi}{2}-1\right)$

$=\dfrac{3\pi}{5}-\dfrac{9}{20}=\dfrac{3}{20}(4\pi-3)$

91. **(b)** $I=\int_0^{\pi}|\cos x|^3\,dx$

$=2\int_0^{\pi/2}\cos^3 x\,dx$

$=\frac{2}{4}\int_0^{\pi/2}(3\cos x+\cos 3x)dx$

[$\because$ cos 3θ = 4cos³ θ – 3cos θ]

$=\frac{1}{2}\left[3\sin x+\frac{\sin 3x}{3}\right]_0^{\pi/2}$

$=\frac{1}{2}\left(3-\frac{1}{3}\right)=\frac{4}{3}$

92. **(a)** $\because f:R\rightarrow R$

and $|f(x)-f(y)|\le 2\cdot|x-y|^{3/2}$

$\Rightarrow \left|\frac{f(x)-f(y)}{x-y}\right|\le 2\sqrt{x-y}$

$\Rightarrow \lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right|\le \lim_{x\to y}2\sqrt{x-y}$

$\Rightarrow |f'(x)|=0$

$\therefore$ $f(x)$ is a constant function.

Given $f(0)=1 \Rightarrow f(x)=1$

Hence, the integral

$\int_0^1 f^2(x)\,dx=\int_0^1 1\,dx=[x]_0^1=1$

93. **(d)** Let, $I=\int_0^{\pi/3}\frac{\tan\theta}{\sqrt{2k\sec\theta}}d\theta$

$=\frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}d\theta$

Let cos θ = t^2

$\therefore$ $\sin\theta\,d\theta=-2t\,dt$

Hence, integral becomes,

$I=\frac{1}{\sqrt{2k}}\int_1^{\sqrt{\frac{1}{2}}}\frac{-2t\,dt}{t}$

$=\sqrt{\frac{2}{k}}\int_{\frac{1}{\sqrt{2}}}^{1}dt$

$=\sqrt{\frac{2}{k}}\left(1-\frac{1}{\sqrt{2}}\right)$

$=\frac{\sqrt{2}-1}{\sqrt{k}}$

$=1-\frac{1}{\sqrt{2}}$ (Given)

$\therefore$ $k=2$

94. **(c)** Let, $I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx$ ...(i)

Using, $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$, we get :

$I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^{-x}}dx$ ...(ii)

Adding (i) and (ii), we get;

$2I=\int_{-\pi/2}^{\pi/2}\sin^2 x\,dx \Rightarrow 2I=2\cdot\int_0^{x/2}\sin^2 x\,dx$

$\Rightarrow 2I=2\times\frac{\pi}{4}\Rightarrow I=\frac{\pi}{4}$

95. **(a)** $f(x)=\int_0^x t(\sin x-\sin t)\cdot dt$

$=\sin x\int_0^x t\cdot dt-\int_0^x t\sin t\cdot dt$

$=\frac{x^2}{2}\sin x+[t\cos t]_0^x+\sin x$

$\Rightarrow f(x)=\frac{x^2}{2}\sin x+x\cos x+\sin x$

$f'(x)=\frac{x^2}{2}\cos x+2\cos x$

$f''(x)=x\cos x-\frac{x^2}{2}\sin x-2\sin x$

$f'''(x)=\cos x-2x\sin x-\frac{x^2}{2}\cos x-2\cos x$

$\therefore f'''(x)+f'(x)=\cos x-2x\sin x$

96. **(a)** Let $I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{x}{1+\sin x}dx$

also let $K=\frac{x}{1+\sin x}$

Multiplying numerator and denominator by $(1-\sin x)$, we get;

$K=\frac{x(1-\sin x)}{1-(\sin x)^2}=\frac{x(1-\sin x)}{(\cos x)^2}$
$= x(1-\sin x)\sec^2 x$
$= x\sec^2 x - x\sin x\sec^2 x = x\sec^2 x - x\tan x\sec x$

Now, $I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x\sec^2 x dx-\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x\sec x\tan x dx$

$$=\left[x\tan x-\int\frac{dx}{dx}\tan x dx\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}-\left[x\sec x-\int\frac{dx}{dx}\sec x dx\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$=\left[x\tan x-\ln|\sec x|\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$-\left[x\sec x-\ln|\sec x+\tan x|\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}+c$$

$$\Rightarrow I=\left\{\left[\frac{3\pi}{4}\tan\frac{3\pi}{4}-\ln\left|\frac{3\pi}{4}\right|\right.\right.$$

$$\left.\left.-\left[\frac{3\pi}{4}\sec\frac{3\pi}{4}-\ln\left|\sec\frac{3\pi}{4}+\tan\frac{3\pi}{4}\right|\right]\right]\right\}$$

$$-\left\{\left[\frac{\pi}{4}\tan\frac{\pi}{4}-\ln\left|\frac{\pi}{4}\right|\right.\right.$$

$$\left.\left.-\left[\frac{\pi}{4}\sec\frac{\pi}{4}-\ln\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right|\right]\right]\right\}$$

$$=\frac{\pi}{2}(\sqrt{2}+1)$$

97. **(d)** Given:

$I_1=\int_0^1 e^{-x}\cos^2 x\,dx;$

$I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx$ and

$I_3=\int_0^1 e^{-x^3}dx$

For $x\in(0,1)$
$\Rightarrow x>x^2$ or $-x<-x^2$
and $x^2>x^3$ or $-x^2<-x^3$

$\therefore e^{-x^2}<e^{-x^3}$ and $e^{-x}<e^{-x^2}$

$\Rightarrow e^{-x}<e^{-x^2}<e^{-x^3}$

$\Rightarrow e^{-x^3}>e^{-x^2}>e^{-x}$

$\Rightarrow I_3>I_2>I_1$

98. **(c)** Let

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx \;.....(1)$$

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4(-x))\left(1+\log\left(\frac{2+\sin(-x)}{2-\sin(-x)}\right)\right)\cdot dx$$

$$=\left[\because\int_a^b f(x).dx=\int_a^b f(a+b-x).dx\right]$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin^4 x)\left(1+\log\left(\frac{2-\sin x}{2+\sin x}\right)\right).dx$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4 x\left(1-\log\left(\frac{2+\sin x}{2-\sin x}\right)\right).dx \;.....(2)$$

After adding equation (1) and (2) we get,

$$2I=2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4 x.dx$$

$$2I=4\int_0^{\frac{\pi}{2}}\sin^4 x.dx$$

$$I=2\int_0^{\frac{\pi}{2}}\sin^4 x.dx=\frac{2\times\frac{3}{2}\times\frac{1}{2}\times\pi}{2\times 2}=\frac{3\pi}{8}$$

[By Gamma function]

99. **(c)** $I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{1+\cos x}$ ...(i)

$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{1-\cos x}$ ...(ii)

Using $\int_a^b f(x)dx=\int_a^b f(a+b-x)\,dx$

Adding (i) and (ii)

$$2I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{2}{\sin^2 x}dx\;;\;I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\text{cosec}^2 x\,dx$$

$$I=-(\cot x)_{\pi/4}^{3\pi/4}=-\left[\cot\frac{3\pi}{4}-\cot\frac{\pi}{4}\right]=2$$

100. **(c)** $I_n=\int\tan^n x\,dx, n>1$

Let $I=I_4+I_6$

$=\int(\tan^4 x+\tan^6 x)dx=\int\tan^4 x\sec^2 x\,dx$

Let tan x = t

$\Rightarrow \sec^2 x\,dx = dt$

$\therefore I = \int t^4 dt = \frac{t^5}{5} + C$

$= \frac{1}{5}\tan^5 x + C \Rightarrow$ On comparing, we have

$a = \frac{1}{5}, b = 0$

101. **(a)** Let $I = \int_1^2 \frac{dx}{((x-1a)^2+3)^{3/2}}$

Let; $x - 1 = \sqrt{3}\tan\theta$

$\Rightarrow dx = \sqrt{3}\sec^2 .d\theta$

$$\Rightarrow I = \int_0^{\pi/6} \frac{\sqrt{3}\sec^2\theta\, d\theta}{\left(\left(\sqrt{3}\tan\theta\right)^2 + \left(\sqrt{3}\right)^2\right)^{3/2}}$$

$$= \frac{1}{3}\int_0^{\pi/6} \frac{\sec^2\theta}{\sec^3\theta} d\theta = \frac{1}{3}\int_0^{\pi/6} \cos\theta\, d\theta$$

$$= \frac{1}{3}[\sin\theta]_0^{\pi/6} = \frac{1}{3}\times\frac{1}{2} = \frac{1}{6}$$

$$= \frac{1}{6} = \frac{k}{k+5} \Rightarrow k + 5 = 6k$$

$$\Rightarrow \boxed{k = 1}$$

102. **(a)** $\int_{\pi/12}^{\pi/4} \frac{\cos 2x}{\left(\frac{1}{\sin 2x}\right)^3} = \int_{\pi/12}^{\pi/4} \cos 2x \times \sin 2x \,.\, \sin^2(2x)\, dx$

$$= \frac{1}{4}\int_{\pi/12}^{\pi/4} \sin 4x\,.\,(1 - \cos 4x)\, dx$$

$$= \frac{1}{4}\left[\int_{\pi/12}^{\pi/4} \sin 4x - \frac{1}{2}\int_{\pi/12}^{\pi/4} \sin 8x\right]$$

$$= \frac{1}{4}\left[-\frac{\cos 4x}{4} + \frac{\cos 8x}{16}\right]_{\pi/12}^{\pi/4} = \frac{1}{4}\left[\frac{15}{32}\right] = \frac{15}{128}$$

103. **(d)** $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$

Dividing by x^{15} in numerator and denominator

$$\int \frac{\frac{2}{x^3} + \frac{5}{x^6} dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

Let $1 + \frac{1}{x^2} + \frac{1}{x^5} = t$

$\Rightarrow \left(\frac{-2}{x^3} - \frac{5}{x^6}\right) dx = dt \Rightarrow \left(\frac{2}{x^3} + \frac{5}{x^6}\right) dx = -dt$

This gives,

$$\int \frac{\frac{2}{x^3} + \frac{5}{x^6} dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} = \int \frac{-dt}{t^3} = \frac{1}{2t^2} + C$$

$$= \frac{1}{2\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^2} + C = \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$$

104. **(d)** $x\int_1^x y(t)\, dt = x\int_1^x ty(t)\, dt + \int_1^x ty(t)\, dt$

Differentiate *w.r.* to *x*.

$$\int_1^x y(t)dt + x[y(x) - y(1)]$$

$$= \int_1^x ty(t)dt + x[xy(x) - y(1)] + xy(x) - y(1)$$

$$\int_1^x y(t)dt = \int_1^x ty(t)dt + x^2 y(x) - y(1)$$

Diff. again *w.r.* to *x*

$y(x) - y(a) = xy(x) - y(a) + 2x\,y(x) + x^2 y^1(x)$

$(1 - 3x)\, y(x) = x^2 y^1(x)$

$$\frac{y^1(x)}{y(x)} = \frac{1 - 3x}{x^2}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1 - 3x}{x^2} \Rightarrow \ln y = -\frac{1}{x} - 3\ln x$$

$$\ln(y x^3) = -\frac{1}{x}$$

$$yx^3 = -e^{-1/x}$$

$$y = \frac{e^{-\frac{1}{x}}}{x^3} \text{ or } y = \frac{ce^{-\frac{1}{x}}}{x^3}$$

105. **(d)** $I = \int_4^{10} \frac{[x^2]}{[x^2 - 28x + 196] + [x^2]} dx$ .....(a)

Use $\int_a^b f(a + b - x)\, dx = \int_a^b f(x)\, dx$

$$I=\int_4^{10}\frac{[(x-14)^2]}{[x^2]+[(x-14)^2]}dx \quad .....(b)$$

(a) + (b)

$$2I=\int_4^{10}\frac{[(x-14)^2]+[x^2]}{[x^2]+[(x-14)^2]}dx$$

$$2I=\int_4^{10}dx \Rightarrow 2I=6 \qquad \Rightarrow I=3$$

106. (b) $2\int_0^1 \tan^{-1}x\,dx=\int_0^1\left(\frac{\pi}{2}-\tan^{-1}(1-x+x^2)\right)dx$

$$2\int_0^1 \tan^{-1}x\,dx=\int_0^1\frac{\pi}{2}dx-\int_0^1\tan^{-1}(1-x+x^2)\,dx$$

$$\int_0^1\tan^{-1}(1-x+x^2)\,dx=\frac{\pi}{2}-2\int_0^1\tan^{-1}x\,dx \quad .....(a)$$

Let, $I_1=\int_0^1\tan^{-1}x\,dx$

$$=\left[(\tan^{-1}x)x\right]_0^1-\int_0^1\frac{1}{1+x^2}x\,dx$$

$$=\frac{\pi}{4}-\int_0^1\frac{x}{1+x^2}dx=\frac{\pi}{4}-\frac{1}{2}\log 2$$

By equation (a)

$$\frac{\pi}{2}-2\left[\frac{\pi}{4}-\frac{1}{2}\log 2\right]=\log 2$$

107. (a) $I=\int_2^4\frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$

$$I=\int_2^4\frac{\log x^2}{\log x^2+\log(6-x)^2}dx \quad ...(i)$$

$$I=\int_2^4\frac{\log(6-x)^2}{\log(6-x)^2+\log x^2}dx \quad ...(ii)$$

Adding (i) and (ii)

$$2I=\int_2^4 dx \;=[x]_2^4=2$$

$I=1$

108. (c) Let $f:R\to R$ be a function such that $f(2-x)=f(e+x)$

Put $x=2+x$ we get

$f(-x)=f(4+x)=f(4-x)$

$\Rightarrow f(x)=f(x+4)$

Hence period is 4

Consider $\int_{10}^{50}f(x)dx=10\int_{10}^{14}f(x)dx=10[5+5]=100$

109. (d) Let $f:(-1,1)\to R$ be a continuous function

Let $\int_0^{\sin x}f(t)dt=\frac{\sqrt{3}}{2}x$

$$f(\sin x).\frac{d}{dx}(\sin x)=\frac{\sqrt{3}}{2}$$

$$\Rightarrow f(\sin x).\cos x=\frac{\sqrt{3}}{2}$$

put $x=\frac{\pi}{3}$

$$f\left(\sin\frac{\pi}{3}\right).\cos\frac{\pi}{3}=\frac{\sqrt{3}}{2}$$

$$f\left(\frac{\sqrt{3}}{2}\right).\frac{1}{2}=\frac{\sqrt{3}}{2}$$

$$f\left(\frac{\sqrt{3}}{2}\right)=\sqrt{3}$$

110. (c) $f\left(\frac{1}{x}\right)=\int_1^{1/x}\frac{\ln t}{1+t}dt$

Let $t=\frac{1}{z}$

$$dt=-\frac{1}{z^2}dz$$

$$f(x)=\int_1^x\frac{\ln z}{z(z+1)}dz$$

$$f(x)+f\left(\frac{1}{x}\right)=\int_1^x\frac{\ln x}{z}dz=\left[\frac{(\ln z)^2}{2}\right]_1^x=\frac{(\ln x)^2}{2}$$

111. (b) Let $I=\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx=\int_0^{\pi}\left|2\sin\frac{x}{2}-1\right|dx$

$$=\int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$$

$$\left[\because \sin\frac{x}{2}=\frac{1}{2}\Rightarrow\frac{x}{2}=\frac{\pi}{6}\right.$$

$$\left.\Rightarrow x=\frac{\pi}{3},\frac{x}{2}=\frac{5\pi}{6}\Rightarrow x=\frac{5\pi}{3}>\pi\right]$$

$$= \left[x + 4\cos\frac{x}{2}\right]_0^{\pi/3} + \left[-4\cos\frac{x}{2} - x\right]_{\pi/3}^{\pi}$$

$$= \frac{\pi}{3} + 4\frac{\sqrt{3}}{2} - 4 + \left(0 - \pi + 4\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)$$

$$= 4\sqrt{3} - 4 - \frac{\pi}{3}$$

112. (d) $F(x) = \int_1^x \frac{e^t}{t}dt$, $x > 0$

Let $I = \int_1^x \frac{e^t}{t+a}dt$

Put $t + a = z \Rightarrow t = z - a;\ dt = dz$
for $t = 1$, $z = 1 + a$
for $t = x$, $z = x + a$

$$\therefore\ I = \int_{1+a}^{x+a} \frac{e^{z-a}}{z}dz$$

$$= e^{-a}\int_{1+a}^{x+a} \frac{e^z}{z}dz \equiv e^{-a}\int_{1+a}^{x+a}\frac{e^t}{t}dt$$

$$I = e^{-a}\left[\int_{1+a}^{1}\frac{e^t}{t}dt + \int_1^{x+a}\frac{e^t}{t}dt\right]$$

$$= e^{-a}\left[-\int_1^{1+a}\frac{e^t}{t}dt + \int_1^{x+a}\frac{e^t}{t}dt\right]$$

$$= e^{-a}[-F(1+a) + F(x+a)]$$

(By the definition of F(x))

$$= e^{-a}[F(x+a) - F(1+a)]$$

113. (a) Let $\int_{-\pi}^{t}(f(x)+x)dx = \pi^2 - t^2$

$$\Rightarrow \int_{-\pi}^{t} f(x)dx + \int_{-\pi}^{t} xdx = \pi^2 - t^2$$

$$\Rightarrow \int_{-\pi}^{t} f(x)dx + \left(\frac{t^2}{2} - \frac{\pi^2}{2}\right) = \pi^2 - t^2$$

$$\Rightarrow \int_{-\pi}^{t} f(x)dx = \frac{3}{2}(\pi^2 - t^2)$$

differentiating with respect to t

$$\frac{d}{dt}\left[\int_{-\pi}^{t} f(x)dx\right] = \frac{3}{2}\frac{d}{dt}(\pi^2 - t^2)$$

$$f(t).\frac{dt}{dt} - f(-\pi)\frac{d}{dt}(-\pi) = -3t$$

$$f(t) = -3t$$

$$f\left(-\frac{\pi}{3}\right) = -3\left(-\frac{\pi}{3}\right) = \pi$$

114. (d) Let $I = \int_0^{\pi}[\cos x]dx$ ...(1)

$$I = \int_0^{\pi}[\cos(\pi - x)]dx = \int_0^{\pi}[-\cos x]dx \quad ...(2)$$

On adding (1) and (2), we get

$$2I = \int_0^{\pi}[\cos x]dx + \int_0^{\pi}[-\cos x]dx$$

$$2I = \int_0^{\pi}[\cos x] + [-\cos x]dx$$

$$2I = \int_0^{\pi} -1dx \qquad (\because [x] + [-x] = -1 \text{ if } x \notin Z)$$

$$2I = -x|_0^{\pi} = -\pi$$

$$\Rightarrow I = \frac{-\pi}{2}$$

115. (c) $P_n = \int_1^e (\log x)^n dx$

put $\log x = t$ then $x = e^t$ and $dx = e^t\,dt$
Also, when $x = 1$, then $t = \log 1 = 0$
and when $x = e$, then $t = \log_e e = 1$

$$\therefore\ P_n = \int_0^1 t^n.e^t\,dt$$

$$\therefore\ P_{10} = \int_0^1 t^{10}e^t dt \text{ and } P_8 = \int_0^1 t^8 e^t dt$$

$$\text{Now, } P_{10} - 90P_8 = \int_0^1 \underset{I}{t^{10}}\,\underset{II}{e^t}\,dt - 90\int_0^1 t^8 e^t dt$$

$$P_{10} - 90\,P_8 = \left[t^{10}e^t\right]_0^1 - 10\int_0^1 t^9 e^t dt - 90\int_0^1 t^8 e^t dt$$

$P_{10} - 90\,P_8$

$$= e - 10\left[t^9\int_0^1 e^t dt - \int_0^1 \frac{d}{dt}(t^9)\int e^t dt\right] - 90\int_0^1 t^8 e^t dt$$

$$P_{10} - 90\,P_8 = e - 10\left[e - 9\int_0^1 t^8 e^t dt\right] - 90\int_0^1 t^8 e^t dt$$

$$P_{10} - 90\,P_8 = e - 10e + 90\int t^8 \cancel{e^t}\,dt - 90\int_0^1 t^8 \cancel{e^t}\,dt$$

$$\therefore\ P_{10} - 90P_8 = -9e$$

116. (c) Let $I = \int_0^{1/2}\frac{\ln(1+2x)}{1+4x^2}dx$ or $\int_0^{1/2}\frac{\ln(1+2x)}{1+(2x)^2}dx$

Put $2x = \tan\theta$

$$\therefore\ \frac{2dx}{d\theta} = \sec^2\theta \text{ or } dx = \frac{\sec^2\theta d\theta}{2}$$

also when $x = 0 \Rightarrow \theta = 0$

and when $x = \frac{1}{2} \Rightarrow \theta = 45°$ or $\frac{\pi}{4}$

$$\therefore \ I = \int_0^{\frac{\pi}{4}} \frac{\ln(1+\tan\theta)}{1+\tan^2\theta} \times \frac{\sec^2\theta d\theta}{2}$$

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\ln(1+\tan\theta)}{1+\tan^2\theta} \times \sec^2\theta d\theta$$

$(\because\ 1 + \tan^2\theta = \sec^2\theta)$

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln(1+\tan\theta) d\theta \quad ...(i)$$

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right] d\theta$$

(Using the property of definite integral)

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln\left[1+\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\times\tan\theta}\right] d\theta$$

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right] d\theta$$

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln\left[\frac{1+\cancel{\tan\theta}+1-\cancel{\tan\theta}}{1+\tan\theta}\right] d\theta$$

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln\left[\frac{2}{1+\tan\theta}\right] d\theta$$

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} [\ln 2 - \ln(1+\tan\theta)] d\theta$$

$$I = \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln 2 . d\theta - \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln(1+\tan\theta) d\theta$$

$$I = \frac{1}{2}\ln 2\theta\Big|_0^{\pi/4} - I \quad \text{(from eq. (i))}$$

$$I + I = \frac{1}{2}\ln 2\left(\frac{\pi}{4}-0\right)$$

$$2I = \frac{1}{2}\times\frac{\pi}{4}\times\ln 2$$

$$2I = \frac{\pi}{8}\ln 2 \ \text{ or } \ I = \frac{\pi}{16}\ln 2$$

117. (a) Since, $y = \int_0^x |t| dt, x \in R$

therefore $\frac{dy}{dx} = |x|$

But from $y = 2x$, $\therefore \frac{dy}{dx} = 2$

$\Rightarrow |x| = 2 \Rightarrow x = \pm 2$

Points $y = \int_0^{\pm 2} |t| dt = \pm 2$

$\therefore$ Equation of tangent is

$y - 2 = 2(x - 2)$ or $y + 2 = 2(x + 2)$

$\Rightarrow$ x-intercept $= \pm 1$.

118. (d) Let, $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$$= \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}\, dx}{1+\sqrt{\tan x}} \quad ...(i)$$

Also, given

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}\, dx}{1+\sqrt{\tan x}} \quad ...(ii)$$

By adding (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} dx$$

$$\Rightarrow I = \frac{1}{2}\left[\frac{\pi}{3}-\frac{\pi}{6}\right] = \frac{\pi}{12},$$

Statement-1 is false

$$\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

It is fundamental property.

Statement -2 is true.

119. (a) Consider

$$\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t})\, dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t})\, dt$$

Let $I = f(x)$ after integrating and putting the limits.

$$f'(x) = \sin^{-1}\sqrt{\sin^2 x}\,(2\sin x\cos x) - 0$$
$$+\cos^{-1}\sqrt{\cos^2 x}\,(-2\cos x\sin x) - 0$$

$\therefore f'(x) = 0 \Rightarrow f(x) = C$ (constant)

Now, we find $f(x)$ at $x = \frac{\pi}{4}$

$$\therefore \ I = \int_0^{1/2} \sin^{-1}\sqrt{t}\, dt + \int_0^{1/2} \cos^{-1}\sqrt{t}\, dt$$

$$= \int_0^{1/2} (\sin^{-1}\sqrt{t} + \cos^{-1}\sqrt{t})\, dt = \int_0^{1/2} \frac{\pi}{2}\, dt = \frac{\pi}{4} = C$$

$\therefore\ f(x) = \frac{\pi}{4}$

$\therefore$ Required integration $= \frac{\pi}{4}$

120. (d) $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ ...(i)

$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-x}} dx$, by replacing x by $\left(\frac{\pi}{2} - \frac{\pi}{2} - x\right)$

$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{2^x . \sin^2 x}{1+2^x} dx$ ...(ii)

Adding equations (i) and (ii), we get

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x\, dx = \frac{1}{2}\int_{-\pi/2}^{\pi/2} (1-\cos 2x)\, dx$$

$$\Rightarrow I = \frac{1}{4}\left[x + \frac{\sin 2x}{2}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{4}\left[\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2}\right)\right]$$

$$\Rightarrow I = \frac{1}{4}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = \frac{\pi}{4}$$

121. (d) Let $I = \int_{7\pi/4}^{7\pi/3} \sqrt{\tan^2 x}\, dx$

$$= \int_{7\pi/4}^{7\pi/3} \tan x\, dx = -\log\cos x\Big|_{7\pi/4}^{7\pi/3}$$

$$= -\left[\log\cos\frac{7\pi}{3} - \log\cos\frac{7\pi}{4}\right]$$

$$= \log\cos\frac{7\pi}{4} - \log\cos\frac{7\pi}{3}$$

$$= \log\left[\frac{\cos\frac{7\pi}{4}}{\cos\frac{7\pi}{3}}\right] = \log\left[\frac{\cos\left(2\pi - \frac{\pi}{4}\right)}{\cos\left(2\pi + \frac{\pi}{3}\right)}\right]$$

$$= \log\left(\frac{\cos\frac{\pi}{4}}{\cos\frac{\pi}{3}}\right) = \log\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}\right)$$

$$= \log\left(\frac{2}{\sqrt{2}}\right) = \log\sqrt{2}.$$

122. (a) $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$

$\Rightarrow 1 = \frac{1}{\sqrt{1+y^2}} \cdot \frac{dy}{dx}$

$\left[\because \text{If } I(x) = \int_{\phi(x)}^{\psi(x)} f(t)\, dt, \text{ then } \frac{dI(x)}{dx} = f\{\psi(x)\}\,.\left\{\frac{d}{dx}\psi(x)\right\} - f\{\phi(x)\}\,.\left\{\frac{d}{dx}\phi(x)\right\}\right]$

$\frac{dy}{dx} = \sqrt{1-y^2}$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{2\sqrt{1+y^2}}\,.\,2y\,.\,\frac{dy}{dx} = \frac{y}{\sqrt{1+y^2}}.\sqrt{1+y^2} = y$$

123. (b, c) $g(x+\pi) = \int_0^{x+\pi} \cos 4t\, dt$

$$= \int_0^x \cos 4t\, dt + \int_x^{\pi+x} \cos 4t\, dt = g(x) + \int_0^{\pi} \cos 4t\, dt$$

(it is clear from graph of cos 4t)

$$\int_x^{\pi+x} \cos 4t\, dt = \int_0^{\pi} \cos 4t\, dt = g(x) + g(\pi) = g(x) - g(\pi)$$

($\because$ From graph of cos 4t, $g(\pi) = 0$)

124. (d) $\int_{-0.9}^{0.9} \left\{[x^2] + \log\left(\frac{2-x}{2+x}\right)\right\} dx$

$$= \int_{-0.9}^{0.9} [x^2]dx + \int_{-0.9}^{0.9} \log\left(\frac{2-x}{2+x}\right) dx$$

$$= 0 + \int_{-0.9}^{0.9} \log\left(\frac{2-x}{2+x}\right) dx$$

Put $x = -x \Rightarrow f(x) = \log\frac{2-x}{2+x}$

and $f(-x) = \log\frac{2+x}{2-x} = -\log\frac{(2-x)}{2+x} = -f(x)$

So, it is an odd function, hence Required integral = 0.

125. (d) Since $\int_0^a [x] = 0$ where $0 \le a \le 1$

$$\therefore \int_0^{0.9} [x - 2[x]]dx = 0$$

126. (a) Let $\frac{d}{dx}G(x)=\frac{e^{\tan x}}{x}, x\in\left(0,\frac{\pi}{2}\right)$

Now, $I=\int_{1/4}^{1/2}\frac{2}{x}e^{\tan \pi x^2}\cdot dx=\int_{1/4}^{1/2}\frac{2\pi x}{\pi x^2}e^{\tan \pi x^2}.dx$

Let $\pi x^2=t \Rightarrow 2\pi x\,dx=dt$

When $x=\frac{1}{2}, t=\frac{\pi}{4}$ and $x=\frac{1}{4}, t=\frac{\pi}{16}$

$\therefore I=\int_{\pi/16}^{\pi/4}\frac{e^{\tan t}}{t}dt=G(t)\Big|_{\frac{\pi}{16}}^{\frac{\pi}{4}}=G\left(\frac{\pi}{4}\right)-G\left(\frac{\pi}{16}\right)$

127. (d) Let $\int_e^x t\,f(t)\,dt=\sin x-x\cos x-\frac{x^2}{2}$

By using Leibnitz rule, we get

$$\frac{d}{dx}\left[\int_e^x t\,f(t)\,dt\right]=\frac{d}{dx}\left[\sin x-x\cos x-\frac{x^2}{2}\right]$$

$\Rightarrow x f(x)-e f(e).\,0=x\sin x-x$

Now, put $x=\frac{\pi}{6}$, we get

$\frac{\pi}{6}.f\left(\frac{\pi}{6}\right)=\frac{\pi}{6}.\sin\frac{\pi}{6}-\frac{\pi}{6}$

$\Rightarrow f\left(\frac{\pi}{6}\right)=\sin\frac{\pi}{6}-1=\frac{1}{2}-1=-\frac{1}{2}$

128. (c) $\int_0^{1.5}x\left[x^2\right]dx=\int_0^1 x[x^2]dx+\int_1^{\sqrt{2}}x\left[x^2\right]dx+\int_{\sqrt{2}}^{1.5}x\left[x^2\right]dx$

$=\int_0^1 x.0\,dx+\int_1^{\sqrt{2}}x dx+\int_{\sqrt{2}}^{1.5}2x dx=0+\left[\frac{x^2}{2}\right]_1^{\sqrt{2}}+\left[x^2\right]_{\sqrt{2}}^{1.5}$

$=\frac{1}{2}(2-1)+(2.25-2)=\frac{1}{2}+0.25$

$=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$

129. (d) $I=\int_0^1\frac{8\log(1+x)}{1+x^2}dx$

Put $x=\tan\theta$,

$\therefore dx=\sec^2\theta d\theta$

$\therefore I=8\int_0^{\pi/4}\frac{\log(1+\tan\theta)}{1+\tan^2\theta}.\sec^2\theta\,d\theta$

$I=8\int_0^{\pi/4}\log(1+\tan\theta)d\theta$ ...(i)

Applying $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

$=8\int_0^{\pi/4}\log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$

$=8\int_0^{\pi/4}\log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right]d\theta=8\int_0^{\pi/4}\log\left[\frac{2}{1+\tan\theta}\right]d\theta$

$=8\int_0^{\pi/4}[\log 2-\log(1+\tan\theta)]d\theta$

$=8\log 2\int_0^{\pi 14}1d\theta-8\int_0^{\pi 14}\log(1+\tan\theta)\,d\theta$

$\text{I}=8\cdot(\log 2)[x]_0^{\pi/4}-8\int_0^{\pi/4}\log(1+\tan\theta)d\theta$

$\text{I}=8\cdot\frac{\pi}{4}\cdot\log 2-\text{I}$ [From equation (i)]

$\Rightarrow 2\text{I}=2\pi\log 2$

$\therefore \text{I}=\pi\log 2$

130. (a) $p'(x)=p'(1-x)$

$\Rightarrow p(x)=-p(1-x)+c$

at $x=0$

$p(0)=-p(1)+c \Rightarrow 42=c$

Now, $p(x)=-p(1-x)+42$

$\Rightarrow p(x)+p(1-x)=42$

Let $I=\int_0^1 p(x)dx$ ...(i)

$\Rightarrow I=\int_0^1 p(1-x)dx$ ...(ii)

Adding eqn. (i) and (ii),

$2I=\int_0^1(42)\,dx \Rightarrow I=21$

131. (c) Let $\text{I}=\int_0^\pi[\cot x]\,dx$ ....(i)

$=\int_0^\pi[\cot(\pi-x)]\,dx=\int_0^\pi[-\cot x]\,dx$ ....(ii)

Adding eq^n s (i) & (ii),

We get

$2\text{I}=\int_0^\pi([\cot x]+[-\cot x])dx$

$=\int_0^\pi(-1)dx$

$[\because [x]+[-x]=-1,\text{ if } x\notin z \text{ and } [x]+[-x]=0, \text{ if } x\in z]$

$=[-x]_0^\pi=-\pi \quad\Rightarrow \text{I}=-\frac{\pi}{2}$

132. (b) We know that $\frac{\sin x}{x} < 1$, for x ∈ (0, 1)

$\Rightarrow \frac{\sin x}{\sqrt{x}} < \sqrt{x}$ on x ∈ (0, 1)

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \sqrt{x} dx = \left[\frac{2x^{3/2}}{3}\right]_0^1$

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \frac{2}{3} \Rightarrow I < \frac{2}{3}$

Also $\frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$ for $x \in (0, 1)$

$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 x^{-1/2} dx = \left[2\sqrt{x}\right]_0^1 = 2$

$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}} dx < 2 \Rightarrow J < 2$

133. (d) $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$

$\therefore \left[\sec^{-1} t\right]_{\sqrt{2}}^{x} = \frac{\pi}{2}$ $\left[\because \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x\right]$

$\Rightarrow \sec^{-1} x - \sec^{-1} \sqrt{2} = \frac{\pi}{2}$

$\Rightarrow \sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \sec^{-1} x = \frac{\pi}{2} + \frac{\pi}{4}$

$\Rightarrow \sec^{-1} x = \frac{3\pi}{4} \Rightarrow x = \sec\frac{3\pi}{4} = \sec\left(\pi - \frac{\pi}{4}\right)$

$\Rightarrow x = -\sec\frac{\pi}{4} \Rightarrow x = -\sqrt{2}$

134. (c) Given that $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where

$f(x) = \int_1^x \frac{\log t}{1+t} dt$

$\therefore F(e) = f(e) + f\left(\frac{1}{e}\right)$

$\Rightarrow F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt$ ...(1)

Let $I = \int_1^{1/e} \frac{\log t}{1+t} dt$

$\therefore$ Put $\frac{1}{t} = z \Rightarrow -\frac{1}{t^2} dt = dz \Rightarrow dt = -\frac{dz}{z^2}$

when $t = 1 \Rightarrow z = 1$ and when $t = 1/e$
$\Rightarrow z = e$

$\therefore I = \int_1^e \frac{\log\left(\frac{1}{z}\right)}{1+\frac{1}{z}} \left(-\frac{dz}{z^2}\right)$

$= \int_1^e \frac{(\log 1 - \log z).z}{z+1} \left(-\frac{dz}{z^2}\right)$

$= \int_1^e -\frac{\log z}{(z+1)} \left(-\frac{dz}{z}\right)$ $[\because \log 1 = 0]$

$= \int_1^e \frac{\log z}{z(z+1)} dz$

$\therefore I = \int_1^e \frac{\log t}{t(t+1)} dt$

[By property $\int_a^b f(t) dt = \int_a^b f(x) dx$]

Now from eqn. (1)

$F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$

$= \int_1^e \frac{t.\log t + \log t}{t(1+t)} dt = \int_1^e \frac{(\log t)(t+1)}{t(1+t)} dt$

$\Rightarrow F(e) = \int_1^e \frac{\log t}{t} dt$

Let $\log t = x$ $\therefore \frac{1}{t} dt = dx$

[when $t = 1, x = 0$ and when $t = e, x = \log e = 1$]

$\therefore F(e) = \int_0^1 x\, dx$ $F(e) = \left[\frac{x^2}{2}\right]_0^1$

$\Rightarrow F(e) = \frac{1}{2}$

135. (b) Let $a = k + h$ where k is an integer such that and $0 \le h < 1$

$\Rightarrow [a] = k$

$\therefore \int_1^a [x] f'(x) dx = \int_1^2 1 f'(x) dx + \int_2^3 2 f'(x) dx +$

$\cdots \int_{k-1}^{k} (k-1) f'(x) dx + \int_{k}^{k+h} k f'(x) dx$

$= \{f(2) - f(1)\} + 2\{f(3) - f(2)\} + 3\{f(4) - f(3)\}$
$+ \ldots\ldots + (k-1)\{f(k) - f(k-1)\} + k\{f(k+h) - f(k)\}$

$= -f(1) - f(2) - f(3) \ldots\ldots - f(k) + kf(k+h)$

$= [a] f(a) - \{f(1) + f(2) + f(3) + \ldots f([a])\}$

136. (c) $I = \int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} [(x+\pi)^3 + \cos^2(x+3\pi)] dx$

Put $x + \pi = t$

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}[t^3+\cos^2 t]dt=2\int_0^{\frac{\pi}{2}}\cos^2 t\,dt$$

[$\because t^3$ is odd and $\cos^2 t$ is even function]

$$=\int_0^{\frac{\pi}{2}}(1+\cos 2t)dt=\frac{\pi}{2}+0$$

137. (d) $I=\int_0^{\pi}xf(\sin x)dx=\int_0^{\pi}(\pi-x)f(\sin x)dx$

$$=\pi\int_0^{\pi}f(\sin x)dx-I\Rightarrow 2I=\pi\int_0^{\pi}f(\sin x)dx$$

$$I=\frac{\pi}{2}\int_0^{\pi}f(\sin x)dx=\pi\int_0^{\pi/2}f(\sin x)dx$$

[$\because \sin(\pi-x)=\sin x$]

$$=\pi\int_0^{\pi/2}f(\cos x)dx$$

138. (b) $I=\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ ...(1)

$$I=\int_3^6\frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}dx \quad ...(2)$$

[$\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx$]

Adding equation (1) and (2)

$$2I=\int_3^6 dx=[x]_3^6=3\Rightarrow I=\frac{3}{2}$$

139. (b) Let $I=\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$ ....(1)

$$=\int_{-\pi}^{\pi}\frac{\cos^2(-x)}{1+a^{-x}}dx$$

$\left[\text{Using }\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$

$$=\int_{-\pi}^{\pi}\frac{a^x\cos^2 x}{1+a^x}dx \quad ...(2)$$

Adding equations (1) and (2) we get

$$2I=\int_{-\pi}^{\pi}\cos^2 x\left(\frac{1+a^x}{1+a^x}\right)dx=\int_{-\pi}^{\pi}\cos^2 x\,dx$$

$$=2\int_0^{\pi}\cos^2 x\,dx\quad [\because f(\pi-x)=f(x)]$$

$$=2\times 2\int_0^{\frac{\pi}{2}}\cos^2 x\,dx=4\int_0^{\frac{\pi}{2}}\sin^2 x\,dx\quad\left[\because f\left(\frac{\pi}{2}-x\right)=f(x)\right]$$

$$\Rightarrow I=2\int_0^{\frac{\pi}{2}}\sin^2 x\,dx=2\int_0^{\frac{\pi}{2}}\left(1-\cos^2 x\right)dx$$

$$\Rightarrow I=2\int_0^{\frac{\pi}{2}}dx-2\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$$

$$\Rightarrow I+I=2\left(\frac{\pi}{2}\right)=\pi\ \Rightarrow I=\frac{\pi}{2}$$

140. (b) $I_1=\int_0^1 2^{x^2}dx,\ I_2=\int_0^1 2^{x^3}dx,$

$$I_3=\int_0^1 2^{x^2}dx,\ I_4=\int_0^1 2^{x^3}dx$$

$\because\ 2^{x^3}<2^{x^2},\ 0<x<1$

$$\Rightarrow \int_0^1 2^{x^2}dx>\int_0^1 2^{x^3}dx\ \Rightarrow\ I_1>I_2$$

and $2^{x^3}>2^{x^2},\ x>1$

$\Rightarrow\ I_4>I_3$

141. (d) $\lim_{x\to 2}\int_0^{f(x)}\frac{4t^3}{x-2}dt=\lim_{x\to 0}\frac{\int_0^{f(x)}4t^3\,dt}{x-2}$

Applying L Hospital rule

$$\lim_{x\to 2}\frac{[4f(x)^3f'(x)]}{1}=4(f(2))^3f'(2)$$

$$=4\times 6^3\times\frac{1}{48}=18$$

142. (d) $f(x)=\frac{e^x}{1+e^x}\Rightarrow f(-x)=\frac{e^{-x}}{1+e^{-x}}=\frac{1}{e^x+1}$

$\therefore f(x)+f(-x)=1\ \forall\ x\in R$

Now $I_1=\int_{f(-a)}^{f(a)}xg\{x(1-x)\}dx$

$$=\int_{f(-a)}^{f(a)}(1-x)g\{x(1-x)\}dx$$

$\left[\text{using }\int_a^b f(x)\,dx\ a=\int_a^b f(a+b-x)dx\right]$

$$\Rightarrow \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx - \int_{f(-a)}^{f(a)} xg(x(1-x)dx$$

$$= I_2 - I_1 \Rightarrow 2I_1 = I_2$$

143. (b) Let $I = \int_0^{\pi} xf(\sin x)dx$ ...(i)

We know that

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx = \int_0^{\pi} (\pi - x) f(\sin x)dx \quad ...(ii)$$

Adding (i) and (ii)

$$\therefore 2I = \pi\int_2^{\pi} f(\sin x)dx = \pi.2\int_0^{\frac{\pi}{2}} f(\sin x)dx$$

$[\because \sin(\pi - x) = \sin x]$

$$\therefore I = \pi\int_0^{\frac{\pi}{2}} f(\sin x)dx \Rightarrow A = \pi$$

Let $\log x = t \Rightarrow e^t = x$

$$\Rightarrow \frac{1}{x}dx = dt \Rightarrow dx = xdt \Rightarrow e^t dt.$$

144. (c) $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}}dx$

$$\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{(\sin x + \cos x)}dx = \int_0^{\frac{\pi}{2}} (\sin x + \cos x)dx$$

$$\left[\because \sin x + \cos x > 0 \text{ if } 0 < x < \frac{\pi}{2}\right]$$

or $I = [-\cos x + \sin x]_0^{\frac{\pi}{2}} = 2$

145. (d) $\int_{-2}^{3} |1-x^2|\,dx = \int_{-2}^{3} |x^2-1|\,dx$

$$\text{Now } |x^2-1| = \begin{cases} x^2-1 & \text{if} \quad x \le -1 \\ 1-x^2 & \text{if} \quad -1 \le x \le 1 \\ x^2-1 & \text{if} \quad x \ge 1 \end{cases}$$

$$\therefore \text{ Integral is } \int_{-2}^{-1} (x^2-1)dx + \int_{-1}^{1} (1-x^2)dx + \int_1^3 (x^2-1)dx$$

$$= \left[\frac{x^3}{3} - x\right]_{-2}^{-1} + \left[x - \frac{x^3}{3}\right]_{-1}^{1} + \left[\frac{x^3}{3} - x\right]_1^3$$

$$= \left(-\frac{1}{3}+1\right) - \left(-\frac{8}{3}+2\right) + \left(2 - \frac{2}{3}\right) + \left(\frac{27}{3} - 3\right) - \left(\frac{1}{3} - 1\right)$$

$$= \frac{2}{3} + \frac{2}{3} + \frac{4}{3} + 6 + \frac{2}{3} = \frac{28}{3}$$

146. (d) $I = \int_0^1 x(1-x)^n dx = \int_0^1 (1-x)(1-1+x)^n dx$

$$= \int_0^1 (1-x)x^n dx = \int_0^1 (x^n - x^{n+1})dx$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1 = \frac{1}{n+1} - \frac{1}{n+2}$$

147. (d) Given that $f'(x) = f(x) \Rightarrow \frac{f'(x)}{f(x)} = 1$

Integrating both side we get

$\log f(x) = x + c \Rightarrow f(x) = e^{x+c}$

$f(0) = 1 \Rightarrow f(x) = e^x$

$\therefore g(x) = x^2 - f(x) = x^2 - e^x$

$$\therefore \int_0^1 f(x)g(x)dx = \int_0^1 e^x(x^2 - e^x)dx$$

$$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$$

$$= \left[x^2 e^x\right]_0^1 - 2\left[xe^x - e^x\right]_0^1 - \frac{1}{2}\left[e^{2x}\right]_0^1$$

$$= e - \left[\frac{e^2}{2} - \frac{1}{2}\right] - 2[e - e + 1] = e - \frac{e^2}{2} - \frac{3}{2}$$

148. (c) $I = \int_a^b xf(x)dx = \int_a^b (a+b-x)f(a+b-x)dx$

We know that

$$\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$= (a+b)\int_a^b f(a+b-x)dx - \int_a^b xf(a+b-x)dx$$

$$= (a+b)\int_a^b f(x)\,dx - \int_a^b xf(x)dx$$

$[\because \text{Given that } f(a+b-x) = f(x)]$

$$2I = (a+b)\int_a^b f(x)dx$$

$$\Rightarrow \quad I = \frac{(a+b)}{2}\int_a^b f(x)dx$$

149. (d) $\lim_{x\to 0}\dfrac{\dfrac{d}{dx}\displaystyle\int_0^{x^2}\sec^2 t\,dt}{\dfrac{d}{dx}(x\sin x)} = \lim_{x\to 0}\dfrac{\sec^2 x^2.2x}{\sin x + x\cos x}$

(by L' Hospital rule)

$$\lim_{x\to 0}\frac{2\sec^2 x^2}{\left(\frac{\sin x}{x}+\cos x\right)} = \frac{2\times 1}{1+1} = 1$$

150. (c) $F(t) = \int_0^t f(t-y)g(y)dy$

$$= \int_0^t e^{t-y}y\,dy = e^t\int_0^t e^{-y}y\,dy$$

$$= e^t\left[-ye^{-y}-e^{-y}\right]_0^t = -e^t\left[ye^{-y}+e^{-y}\right]_0^t$$

$$= -e^t\left[t\,e^{-t}+e^{-t}-0-1\right] = -e^t\left[\frac{t+1-e^t}{e^t}\right]$$

$$= e^t-(1+t)$$

151. (b) $\int_{-\pi}^{\pi}\dfrac{2x\,(1+\sin x)}{1+\cos^2 x}dx$

$$= \int_{-\pi}^{\pi}\frac{2x\,dx}{1+\cos^2 x} + 2\int_{-\pi}^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$

$$= 0+4\int_0^{\pi}\frac{x\sin x\,dx}{1+\cos^2 x};$$

We know that

$$\because \int_{-a}^{a} f(x)dx = 0, \text{ if } f(x) \text{ is odd.}$$

$$= 2\int_0^a f(x)dx, \text{ if } f(x) \text{ is even}$$

$$I = 4\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$$

$$I = 4\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow I = 4\pi\int_0^{\pi}\frac{\sin x\,dx}{1+\cos^2 x} - 4\int\frac{x\sin x\,dx}{1+\cos^2 x}$$

$$\Rightarrow 2I = 4\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$

put $\cos x = t \Rightarrow -\sin x\,dx = dt$

when $x = 0, t = 1$ and when $x = \pi, t = -1$

$$\therefore I = -2\pi\int_1^{-1}\frac{1}{1+t^2}dt = 2\pi\int_{-1}^{1}\frac{1}{1+t^2}dt$$

$$= 2\pi\left[\tan^{-1}t\right]_{-1}^{1} = 2\pi\left[\tan^{-1}1-\tan^{-1}(-1)\right]$$

$$= 2\pi\left[\frac{\pi}{4}-\left(\frac{-\pi}{4}\right)\right] = 2\pi.\frac{\pi}{2} = \pi^2$$

152. (d) We know that $[x]$ is greatest integer function less than equal to x

$$\therefore \int_0^2\left[x^2\right]dx = \int_0^1\left[x^2\right]dx + \int_1^{\sqrt{2}}\left[x^2\right]dx + \int_{\sqrt{2}}^{\sqrt{3}}\left[x^2\right] + \int_{\sqrt{3}}^{2}\left[x^2\right]dx$$

$$= \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx$$

$$= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$$

$$= \sqrt{2}-1+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}$$

$$= 5-\sqrt{3}-\sqrt{2}$$

153. (b) $I_n + I_{n+2} = \int_0^{\pi/4}\tan^n x(1+\tan^2 x)dx$

$$= \int_0^{\pi/4}\tan^n x\sec^2 x\,dx = \left[\frac{\tan^{n+1}x}{n+1}\right]_0^{\pi/4}$$

$$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1}\right]$$

$$= \frac{1-0}{n+1} = \frac{1}{n+1}$$

$$\therefore I_n + I_{n+2} = \frac{1}{n+1} \Rightarrow \lim_{n\to\infty} n\,[I_n + I_{n+2}]$$

$$= \lim_{n\to\infty} n.\frac{1}{n+1} = \lim_{n\to\infty}\frac{n}{n+1} = \lim_{n\to\infty}\frac{n}{n\left(1+\frac{1}{n}\right)} = 1$$

154. (a) $I = \int_0^{10\pi}|\sin x|\,dx = 10\int_0^{\pi}|\sin x|\,dx$

$[\because \sin(10\pi - x) = \sin x]$

$$= 10\int_0^{\pi}\sin x\,dx$$

$\because \sin x > 0$, for $0 < x < \pi$.

as $\sin(\pi - x) = \sin x$

$$I = 20\int_0^{\pi/2}\sin x\,dx = 20[-\cos x]_0^{\pi/2} = 20$$

155. (a) $F(x)=\int_1^x t^2 g(t)dt$

Differentiate by using Leibnitz's rule, we get

$F'(x)=x^2g(x)=x^2\int_1^x f(u)du$ ...(i)

At $x=1$,

$F'(1)=1\int_1^1 f(u)du=0$

Now, differentiate eqn (i)

$F''(x)=x^2f(x)-2x\int_1^x f(u)du$

At $x=1$,

$F''(1)=1.f(1)-2\times1.\int_1^1 f(u)du$

$=f(1)-2\times0=f(1)$

$F''(1)=3$

Then, for $F'(1)=0, F''(1)=3>0$

Hence, $x=1$ is a point of local minima.

156. (a) $\lim_{n\to\infty}\frac{(n+1)^{\frac13}+(n+2)^{\frac13}+...+(n+n)^{\frac13}}{n(n)^{\frac13}}$

$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{(n+r)^{\frac13}}{n.n^{\frac13}}$

$=\int_0^1(1+x)^{\frac13}dx$ $\left[\because \frac{r}{n}\to x \text{ and } \frac{1}{n}\to\frac{d}{x}\right]$

$=\left[\frac34(1+x)^{\frac43}\right]_0^1=\frac34(2)^{\frac43}-\frac34$

157. (d) Let $L=\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}=\int_0^2\frac{dx}{1+x^2}$

$\left[\because \frac{r}{n}\to x, \frac{1}{r}\to dx\right]$

$=\left[\tan^{-1}x\right]_0^2$

$=\tan^{-1}2$

158. (a) $\lim_{n\to\infty}\frac{\frac{1}{(a+1)}.n^{a+1}+a_1n^a+a_2n^{a-1}+.......}{(n+1)^{a-1}.n^2\left(a+\frac{1+\frac1n}{2}\right)}=\frac{1}{60}$

$\Rightarrow \lim_{n\to\infty}\frac{\left(\frac1n\right)^a+\left(\frac2n\right)^a+.......+\left(\frac nn\right)^a}{(n+1)^{a-1}\left[n^2a+\frac{n(n+1)}{2}\right]}=\frac{1}{60}$

$=\frac{\lim_{n\to\infty}\frac1n\sum_{r=1}^{n}\left(\frac rn\right)^a}{\left(1+\frac1n\right)^{a-1}\left[a+\frac12\left(1+\frac1n\right)\right]}=\frac{1}{60}$

$=\frac{\int_0^1 x^a dx}{\left(a+\frac12\right)}=\frac{1}{60}=\frac{\frac{1}{a+1}}{a+\frac12}=\frac{1}{60}$

$\Rightarrow \frac{\frac{1}{a+1}}{\left(a+\frac12\right)}=\frac{1}{60}$

$\Rightarrow (a+1)(2a+1)=120$

$\Rightarrow 2a^2+3a-119=0$

$\Rightarrow 2a^2+17a-14a-119=0$

$\Rightarrow (a-7)(2a+17)=0$

$\Rightarrow a=7, -\frac{17}{2}$

159. (d) $y=\lim_{n\to\infty}\left(\frac{(n+1)(n+2)...3n}{n^{2n}}\right)^{\frac1n}$

$\ln y=\lim_{n\to\infty}\frac1n\ln\left(1+\frac1n\right)\left(1+\frac2n\right)....\left(1+\frac{2n}{n}\right)$

$\ln y=\lim_{n\to\infty}\frac1n\left[\ln\left(1+\frac1n\right)+\ln\left(1+\frac2n\right)+....+\ln\left(1+\frac{2n}{n}\right)\right]$

$=\lim_{n\to\infty}\frac1n\sum_{r=1}^{2n}\ln\left(1+\frac rn\right)=\int_0^2\ln(1+x)dx$

Let $1+x=t\Rightarrow dx=dt$

when $x=0,\ t=1$

$x=2,\ t=3$

$\ln y=\int_1^3\ln t\,d\,t=[t\ln t-t]_1^3=\ln\left(\frac{3^3}{e^2}\right)=\ln\left(\frac{27}{e^2}\right)$

$\Rightarrow y=\frac{27}{e^2}$

160. (a) Let $f(x) = \int \frac{dx}{\sin^6 x}$

$f(x) = \int \text{cosec}^6 x\, dx$

From reduction formula, we have

$$I_n = \int \text{cosec}^n x\, dx = -\frac{\text{cosec}^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} I_{n-2}$$

$$\therefore f(x) = -\frac{\text{cosec}^4 x \cot x}{5} + \frac{4}{5}\left[\frac{-\text{cosec}^2 x \cot x}{3} + \frac{2}{3} I_2\right]$$

$$= -\frac{\text{cosec}^4 x \cot x}{5} - \frac{4}{15}\text{cosec}^2 x.\cot x + \frac{8}{15}[-\cot x]$$

$$= \frac{-(1+\cot^2 x)^2.\cot x}{5} - \frac{4}{15}(1+\cot^2 x)\cot x - \frac{8}{15}(-\cot x) \quad (\because \text{cosec}^2 x = 1 + \cot^2 x)$$

$$= \frac{-1}{5}\left[1 + \cot^4 x + 2\cot^2 x\right]\cot x - \frac{4}{15}\left[\cot x + \cot^3 x\right] - \frac{8}{15}\cot x$$

$$= \frac{-1}{5}[\cot x + \cot^5 x + 2\cot^3 x] \quad \frac{-4}{15}\cot x - \frac{4}{15}\cot^3 x - \frac{8}{15}\cot x$$

$$= \frac{-15}{15}\cot x - \frac{\cot^5 x}{5} - \frac{10}{15}\cot^3 x$$

$$= \frac{-\cot^5 x}{5} - \frac{2}{3}\cot^3 x - \cot x$$

It is a polynomial of degree 5 in cot x.

161. (d) $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \frac{3}{n^2}\sec^2\frac{9}{n^2} + ... + \frac{1}{n}\sec^2 1\right]$ is equal to

$$\lim_{n\to\infty}\frac{r}{n^2}\sec^2\frac{r^2}{n^2} = \lim_{n\to\infty}\frac{1}{n}.\frac{r}{n}\sec^2\frac{r^2}{n^2}$$

$\Rightarrow$ Given limit is equal to value of integral

$$\int_0^1 x\sec^2 x^2 dx$$

or $\frac{1}{2}\int_0^1 2x\sec\ x^2 dx = \frac{1}{2}\int_0^1 \sec^2 t dt$ [put $x^2 = t$]

$$= \frac{1}{2}(\tan t)_0^1 = \frac{1}{2}\tan 1.$$

162. (b) $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}}$ [Using definite integrals as limit of sum]

$$= \int_0^1 e^x dx = e - 1$$

163. (a) $\lim_{n\to\infty}\frac{1^4 + 2^4 + 3^4 + ...n^4}{n^5} - \lim_{n\to\infty}\frac{1^3 + 2^3 + 3^3 + ... + n^3}{n^5}$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^4 - \lim_{n\to\infty}\frac{1}{n}.\lim_{n\to\infty}\frac{1}{n}\left(\frac{r}{n}\right)^3$$

$$= \int_0^1 x^4\, dx - \lim_{n\to\infty}\frac{1}{n}\times\int_0^1 x^3\, dx = \left[\frac{x^5}{5}\right]_0^1 - 0 = \frac{1}{5}$$

164. (a) We have $\lim_{n\to\infty}\frac{1^p + 2^p + ... + n^p}{n^{p+1}}$;

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^p}{n^p\cdot n} = \int_0^1 x^p dx = \left[\frac{x^{p+1}}{p+1}\right]_0^1 = \frac{1}{p+1}$$